

RESEARCH ARTICLE

Mobile-Supported Blended Learning for Fractions: Enhancing Conceptual and Procedural Knowledge in Primary School Students

Maria Arvanitaki^{1*} Nicholas Zaranis¹ Michalis Linardakis¹ Michail Kalogiannakis²

¹ Department of Preschool Education, University of Crete, Crete, Greece

² Department of Special Education, University of Thessaly, Thessaly, Greece

(Check for updates

Correspondence to: Maria Arvanitaki, Department of Preschool Education, Faculty of Education, University of Crete, Crete 74100, Greece; Email: arvanitakm@sch.gr

Received: March 20, 2025; **Accepted:** June 18, 2025; **Published:** June 23, 2025.

Citation: Arvanitaki, M., Zaranis, N., Linardakis, M., & Kalogiannakis, M. (2025). Mobile-Supported Blended Learning for Fractions: Enhancing Conceptual and Procedural Knowledge in Primary School Students. Advances in Mobile Learning Educational Research, 5(2), 1449-1462. https://doi.org/10.25082/AMLER.2025.02.003

Copyright: © 2025 Maria Arvanitaki et al. This is an open access article distributed under the terms of the Creative Commons Attribution-Noncom-mercial 4.0 International License, which permits all noncommercial use, distribution, and reproduction in any medium, provided the original author and source are credited.



Abstract: This study explores the impact of a blended learning approach on fifth-grade students' conceptual and procedural knowledge of fractions. A quasi-experimental design was implemented with 130 fifth grade students from public primary schools in socio-economically homogeneous areas of Heraklion, Crete (control group: n = 64; experimental group: n = 66). The intervention combined mobile-accessible H5P interactive tasks designed on e-Me learning platform, featuring dynamic environments in Geogebra, with hands-on activities using manipulatives and student-constructed models. Emphasis was placed on collaborative learning and verbal articulation of reasoning. Wilcoxon signed-rank tests revealed significant within-group improvements in both groups. However, the experimental group showed greater gains. Mann-Whitney U tests confirmed that improvements in conceptual and procedural knowledge were significantly higher in the experimental group (p = 0.003 and p = 0.008, respectively), with moderate effect sizes. These findings suggest that the blended learning approach substantially supports fraction learning by bridging conceptual and procedural aspects of knowledge. The use of browser-based, open-source software proved effective in creating personalised, engaging learning experiences. This study contributes to the growing discussion on technology-enhanced fraction learning by presenting a flexible, learner-centered approach that empowers teachers to design contextually responsive fraction learning experiences.

Keywords: blended learning, mobile learning, primary education, conceptual knowledge, procedural knowledge

1 Introduction

Fractions remain one of the most complex domains in mathematics education. Students often exhibit persistent misconceptions about the meaning of fractions (Schneider & Siegler, 2010; Siegler et al., 2013; Vamvakoussi & Vosniadou, 2010), particularly when transitioning from whole number reasoning (Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004; Siegler & Lortie-Forgues, 2015). Research distinguishes fraction learning between two fundamental components: conceptual knowledge, which refers to understanding the underlying structures of fractions (part-whole, ratio, quotient, measure, operator), and procedural knowledge, which involves the ability to carry out arithmetic operations with fractions accurately and efficiently (Byrnes & Wasik, 1991; Hiebert & Lefevre, 1986; Lamon, 2007; Rittle-Johnson & Schneider, 2015). These two types of knowledge are highly interrelated that weaknesses in one domain can hinder the development of the other (Byrnes & Wasik, 1991; Hecht & Vagi, 2010; Rittle-Johnson & Alibali, 1999). For example, procedural fluency without an underlying conceptual foundation may result in fragile, temporal learning that does not transfer to different contexts, while conceptual understanding without procedural practice may not be translated into effective problem solving (Rittle-Johnson et al., 2001; Star, 2005).

Responding to the need for coherence between conceptual and procedural learning, several researchers have proposed instructional methods that promote the simultaneous development of both knowledge types (Behr et al., 1992; Charalambous & Pitta-Pantazi, 2007; Kieren, 1993; Lamon, 2007; 2020). These include approaches grounded in constructivism, multiple representations, and dynamic visualization (Charalambous & Pitta-Pantazi, 2007; Goodwin, 2012; Stafylidou & Vosniadou, 2004). Activities that encourage students to build their own representations, justify their reasoning, and connect symbolic notation with real-world contexts have been shown to foster more robust understandings of fractions (Escuder & Furner, 2011;

Koleza, 2000; Niemi, 1996). Additionally, experiential and collaborative activities, such as using manipulatives, drawing models, and sharing explanations during classroom discussions, can enhance students' conceptual depth bridging their intuitive and formal knowledge (Arcavi, 2003, 2007; Doğan & Tertemiz, 2020; Lamon, 2020; Moss, 2005; Papadakis, Alexandraki & Zaranis, 2022; Post et al., 1993; Van de Walle et al., 2016; Wilkie & Roche, 2022; Zhang et al., 2014).

Educational research on the blended learning approach, which combines experiential teaching with digital tasks, has highlighted important benefits, including flexibility, engagement, active participation and the reflective use of digital tools (Ampartzaki et al., 2024; Cleveland-Innes & Wilton, 2018; Jailani et al., 2025; Laskaris et al., 2017; 2019; Motteram & Sharma, 2009; Papadakis et al., 2020; Powell et al., 2015). In mathematics education, such approaches are gaining ground as promising frameworks for promoting both engagement and deeper learning (Ekeh et al., 2021; Fazal & Bryant, 2019; Iroko & Olaoye, 2021; Indrapangastuti et al., 2021; Kundu et al., 2024; Obot, 2023; Yaghmour, 2016).

Within this blended context, mobile-supported learning environments further enhance engagement and flexibility (Attard & Orlando, 2014; Goodwin, 2012; Green & Hannon, 2006; Kalogiannakis & Papadakis, 2020; UNESCO, 2013), while promoting cooperation and teamwork among students (Clarke & Svanaes, 2014; Papadakis, 2020) as well as facilitating task rotation (Arvanitaki & Zaranis, 2020; Attard & Curry, 2012). In contrast to studies that focus mainly on the use of applications with specific limited features (Arvanitaki & Zaranis, 2020; Goodwin & Highfield, 2012; Papadakis et al., 2021), access to browser-based platforms, particularly open-source software, enables teachers to create customized and personalized content, based on the learning objectives and their students' specific needs (Chilivumbo, 2015; Gazzawe et al., 2022).

Building on this rationale, this study investigates the impact of a blended learning approach that combines browser-based mobile access to digital tasks with hands-on experiential activities to support fraction learning in primary education. The intervention aims to strengthen both conceptual and procedural aspects of fraction knowledge through an integrated pedagogical design that is grounded in accessibility, adaptability, and active student engagement. To address these aim, the present study was guided by the following research questions:

(1) Did control group students show statistically significant improvement in their conceptual and procedural fraction knowledge following traditional instruction?

(2) Did experimental group students show statistically significant improvement in their conceptual and procedural fraction knowledge following the blended learning intervention?

(3) Did the experimental group demonstrate significantly greater improvement in conceptual and procedural fraction knowledge compared to the control group?

2 Materials and methods

2.1 Research Design

A quasi-experimental pretest-posttest design with two groups was implemented to investigate the impact of a blended learning approach on fifth-grade students' fraction conceptual and procedural knowledge. The design allowed for the comparison of learning outcomes between the experimental group exposed to the blended intervention and the control group receiving regular instruction. This design is widely used in educational research to evaluate instructional interventions in authentic classroom contexts (Cohen et al., 2018; Creswell, 2016).

The intervention was designed on the basis of the blended learning method, so it combined browser-based digital tasks accessed via tablets with hands-on experiential activities, aiming to foster deeper and more connected fraction learning.

2.2 Participants

The study involved 130 fifth-grade students from public primary schools in socioeconomically homogeneous areas of Heraklion, Crete. Participants were divided into two groups, the experimental (n = 66) and the control (n = 64) group. Experimental group received the blended learning intervention, while control group received traditional instruction aligned with the official curriculum.

The study adhered to all necessary ethical guidelines for research with children. Approval was obtained to conduct the survey in the primary schools of the Municipality of Heraklion for the school year 2024-2025, and parental consent was obtained for all participants. Students

were informed about the study in age-appropriate language and participated voluntarily, with the option to withdraw at any time. The data collection process followed ethical considerations, ensuring the confidentiality and anonymity of the participants' responses.

2.3 Intervention Materials and Procedure

The teaching intervention lasted six weeks for both the control and experimental groups and took place during the second phase of the study, following the pretest. Both groups covered the same fraction content aligned with the Greek curriculum. Instruction occurred during regular class hours, taught by the students' usual teachers to maintain authentic classroom conditions. Thus, instructional time and core content were equivalent across groups, differing only in the instructional method. The control group teachers followed the conventional teaching method, while the experimental group teachers received specific training and materials to implement the blended learning approach. To ensure consistent implementation and monitor fidelity, teachers completed a structured checklist report at the end of each week, which covered key components of the intervention, such as the completion of digital tasks and hands-on activities. Additionally, teachers reported any deviations from the planned activities and any technical or other difficulties they encountered. They also provided observations about student engagement.

The intervention was based on a blended learning approach that integrated both digital and experiential components. Digital content was accessed through tablets that were shared among 2-3 students, ensuring equal access and collaboration (Figure 1). Digital tasks were created using open, browser-compatible platforms, such as H5P (hosted on the e-me, the official digital learning platform of the Greek Ministry of Education) and Geogebra. These were embedded within an *e-me blog* environment as a digital organizer for structured access to the learning content. As illustrated in Figure 2, digital activities included interactive representations of fractional quantities (e.g., dynamic pie charts, number lines, visual models) and their associated operations. They were designed to promote self-paced exploration, immediate feedback, and collaboration through shared tablet use. The digital content was designed according to Mayer's Cognitive Theory for Multimedia Learning (2009; 2014), incorporating both basic and advanced principles to optimize cognitive processing. For example, the Coherence Principle guided the exclusion of extraneous content, while the Signaling Principle was applied through emphasis using bold text, color and summaries. The Spatial Contiguity and Segmenting Principles were addressed by placing related text and visuals close together and presenting content in manageable segments with navigational control. According to Multimedia Principle, static and dynamic visuals were paired with text, while a conversational tone and familiar contexts supported Personalization and Voice Principles. Additionally, Worked Examples, Immediate Feedback, and Multiple Representations of fractions were used to reinforce fraction understanding.

Experiential activities included manipulatives, like tangram pieces and fraction tiles (Figure 3), student-constructed models, tasks from the official math textbook and activities promoting reasoning. A specialized 'Manipulation Material Notebook' was developed to guide and support the implementation of experiential tasks.



Figure 1 Collaborative Engagement on Digital Content via tablets



Figure 2 Dynamic Activity in Geogebra



Figure 3 Experiential activities involving manipulatives

The intervention followed the Rotation Model of blended learning (Staker & Horn, 2012), where students engaged alternately in digital and hands-on collaborative activities within each lesson. The two modalities were purposefully combined to reinforce learning depending on the context. In some cases, the experiential activities with physical materials served as an alternative to the digital ones allowing students to choose their preferred mode, fostering agency and engagement. In other cases, hands-on activities extended the digital activities by enhancing students' visualization or reasoning skills.

Particular emphasis was placed on sequencing of activities to provide students with rich, interconnected learning experiences that highlight the conceptual structures of fractions and their relationship to operations. Within this rationale, dynamic and constructive activities were introduced first to establish strong conceptual foundations. These visual and constructive tasks enabled students to grasp the underlying fraction structures before engaging with procedures, ensuring that symbolic operations were grounded in meaningful understanding. Next, practice problem solving tasks were presented, allowing students to apply procedures in ways that were meaningfully connected to their prior understanding. Following these procedural tasks, metacognitive activities involving argumentation and reflection were incorporated, encouraging students to justify their reasoning and articulate alternative strategies. This sequence supported the internalization of concepts and reinforced the connection between conceptual and procedural fluency.

The intervention was guided by Kieren's (1976; 1980) distinction into five conceptual subconstructs of fractions (part-whole, ratio, quotient, measure, operator) in conjunction with the theoretical model of Behr, Lesh, Post and Silver (1983), which links the above conceptual structures to the operations of fractions, equivalence and problem solving. In the present research we focused on four of the above conceptual structures (part-whole, quotient, measure, operator), as these align with the official 5th-grade mathematics curriculum and learning objectives. Also included are the operations between fractions (addition, subtraction, multiplication, division), concepts of equivalence and comparison of fractions, improper fractions, mixed numbers and reduction to the fractional unit, which are teaching objectives of the fractions unit in 5th grade.

2.4 Instruments

To assess learning outcomes, the following instruments were developed by the research team:

A Fraction Conceptual Knowledge Pretest and Posttest, designed to assess knowledge of four core conceptual structures of fractions relevant to the fifth-grade curriculum: part-whole, quotient, measure and operator.

A Fraction Procedural Knowledge Pretest and Posttest, designed to evaluate students' ability to perform fraction operations (addition, subtraction, multiplication, division) and tasks involving equivalence, comparison, conversions between improper fractions and mixed numbers and reduction to the fractional unit.

Both tests consisted of closed-ended and short-answer questions with two levels of difficulty, basic and advanced. Each test had a total score ranging from 0 to 8, with points distributed individual items and their sub-questions based on accuracy and completeness.

The tests were created using elements and principles from the Fractional Knowledge Assessment (FKA) research tool, as described by Kalra, Hubbard and Matthews (2020). We also drew on exercises and criteria proposed by other researchers working on this topic (Behr et al., 1983; Charalambous & Pitta-Pantazi, 2007; Kieren, 1980; 1993; Lamon, 2007; 2020; Niemi, 1996).

Although a detailed item-to-sub-construct mapping is not included here, the test content was reviewed by mathematics education experts to confirm its alignment with the constructs, the operations and the fifth-grade curriculum objectives.

Prior to the main study, the tests were pilot tested with eight fifth-grade students (n = 8) to evaluate their structure, clarity, difficulty and timing. Feedback from this pilot led to improvements, supporting the instruments' validity and reliability for this age group.

3 Results

This study examines how a blended learning approach affects elementary school pupils' comprehension of fractions, emphasizing conceptual and procedural knowledge. The intervention combined digital tools and experiential learning to simultaneously strengthen conceptual and procedural fluency in fractions. In order to create interesting and contextualized learning opportunities, the digital component integrated interactive H5P-based activities and dynamic environments, into an e-me blog that was accessible on mobile devices.

Prior to the main analyses, we assessed the normality of the distributions for all pre-test, post-test and improvement scores in both domains of fraction knowledge. The Shapiro-Wilk test showed significant deviations from normality in all cases for at least one of the two groups. Specifically, in conceptual knowledge: pre-test (control group = 0.963, p = 0.050; experimental group = 0.929, p = 0.001); post-test (control group = 0.936, p = 0.003; experimental group =

0.965, p = 0.062); and improvements (control group = 0.959, p = 0.031; experimental group = 0.983, p = 0.484). Similar discrepancies were present in procedural knowledge: pretest (control group = 0.936, p = 0.002; experimental group = 0.848, p < 0.001); posttest (control group = 0.951, p = 0.013; experimental group = 0.938, p = 0.003); and improvements (control group = 0.954, p = 0.018; experimental group = 0.956, p = 0.019).

Therefore, all subsequent analyses were conducted using non-parametric tests, the Mann-Whitney U test and the Wilcoxon signed-rank test for between-group and within-group comparisons, respectively.

Before interpreting learning outcomes, we tested for group equivalence at baseline to ensure comparability. Gender distribution between the control and experimental groups was assessed using a chi-square test, which confirmed no statistically significant difference between the two groups ($\chi^2 = 0.280$, p = 0.597 > 0.050), indicating a balanced gender distribution.

To determine equivalence in prior knowledge, Mann-Whitney U tests were conducted to compare the groups' pretest performance (Table 1 and 2).

	Statistic	р	Effect Size (Rank biserial correlation)
Conceptual knowledge pretest	1819.000	0.172	0.139
Procedural knowledge pretest	1998.500	0.596	-0.054

Table 1 Mann-Whitney U Test for Pretest

 Table 2
 Medians (and IQR) for Students Performance in the Pretests by Group

	Ν	Control	Experimental
Conceptual knowledge pretest	64	1.875 (1.000)	2.250 (2.125)
Procedural knowledge pretest	66	0.750 (0.813)	0.750 (1.000)

In the conceptual domain the results indicate that the experimental group had a slightly higher median score in conceptual knowledge (median = 2.250, IQR = 2.125) compared to control group (median = 1.875, IQR = 1.000). However, this difference was not statistically significant (U = 1819.000, p = 0.172) and the effect size was small (r = 0.139).

In the procedural domain, both groups had the same median score of 0.750, although the control group showed greater variability (IQR = 0.813) than the experimental group (IQR = 1.000). Again, the difference was not statistically significant (U = 1998.500, p = 0.596), with a negligible effect size (r = -0.054).

These findings confirm that the groups were equivalent in terms of their prior knowledge before the intervention in both conceptual and procedural fraction knowledge.

3.1 Control Group: Pretest-Posttest Comparison

To address the first research question, we examined whether control group students showed statistically significant improvement in their conceptual and procedural fraction knowledge following traditional instruction. Wilcoxon's signed-rank test was conducted for each type of knowledge comparing pretest and posttest scores.

Table 3 and Figure 4 illustrate clear improvements in both knowledge domains after instruction. Most students achieved higher posttest scores in conceptual knowledge with only 7 out 64 students (approx. 10%) showing a decline. All students improved their procedural knowledge (Table 4). The results, presented in Table 5, indicate statistically significant improvement and a strong effect size for both conceptual (z = -6.175, p < .001, r = 0.901) and procedural knowledge (z = -6.957, p < .001, r = 1.000).

These results demonstrate significant gains in both conceptual and procedural knowledge in the control group.

Table 3	Medians	(and IQR) for	the Control Group	5 Students Perf	ormance
---------	---------	---------------	-------------------	-----------------	---------

	Pretest	Posttest
Conceptual knowledge	1.875 (1.000)	4.000 (3.500)
Procedural knowledge	0.750 (0.813)	5.000 (3.000)



Figure 4 Medians of Pretest and Posttest Scores for the Control Group

		Ν	Mean Rank	Sum of Ranks
	Negative Ranks	7	13.790	96.500
Conceptual knowledge	Positive Ranks	55	33.750	1856.500
posttest-pretest	Ties	2		
	Total	64		
	Negative Ranks	0	0.000	0.000
Procedural knowledge	Positive Ranks	64	32.500	2080.000
posttest-pretest	Ties	0		
• •	Total	64		

 Table 4
 Wilcoxon Signed-Rank Results for the Control Group

Table 5	Wilcoxon Signed Rank Test Statistics and Effect Sizes
---------	---

Test Statistic	Z	р	Effect Size (Rank biserial correlation)
Conceptual knowledge posttest-pretest	-6.175	0.000	0.901
Procedural knowledge posttest-pretest	-6.957	0.000	1.000

3.2 Experimental Group: Pretest-Posttest Comparison

To address the second research question, we examined whether experimental group students showed statistically significant improvement in their conceptual and procedural fraction knowledge following blended learning intervention. Wilcoxon's signed-rank test was conducted comparing pretest and posttest scores for each domain.

Table 6 and Figure 5 illustrate clear improvements in both conceptual and procedural knowledge after instruction. Median scores increased substantially from pretest to posttest in both domains. Notably, the interquartile range for conceptual knowledge decreased, showing more consistent performance among students following the intervention.

 Table 6
 Medians (and IQR) for the Experimental Group Students Performance

	Pretest	Posttest
Conceptual knowledge	2.250 (2.125)	6.000 (1.440)
Procedural knowledge	0.750 (1.000)	5.750 (2.130)

As shown in Table 7, nearly all students demonstrated improvement in conceptual understanding, with only 2 out of 66 students (approx. 3%) scoring lower than in pretest. In procedural knowledge, all students improved. These gains were statistically significant with a very strong effect size for conceptual (z = -7.028, p < .001, r = 0.995) and procedural knowledge (z = -7.066, p < .001, r = 1.000; Table 8). These findings imply that the experimental group achieved significant and consistent improvements in both knowledge domains.



Figure 5 Medians of Pretest and Posttest Scores for the Experimental Group

 Table 7
 Wilcoxon Signed-Rank Results for the Experimental Group

		Ν	Mean Rank	Sum of Ranks
Conceptual knowledge posttest-pretest	Negative Ranks Positive Ranks Ties Total	2 64 0 66	3.000 34.550	6.000 2205.000
Procedural knowledge posttest-pretest	Negative Ranks Positive Ranks Ties Total	0 66 0 66	0.000 33.500	0.000 2011.000

Table 8 Wilcoxon Signed Rank Test Statistics and Effect Sizes

Test Statistic	Ζ	р		Effect Size
Conceptual knowledge posttest-pretest	-7.028	$0.000 \\ 0.000$	Rank biserial correlation	0.995
Procedural knowledge posttest-pretest	-7.066		Rank biserial correlation	1.000

3.3 Between-Group Comparison of Improvement

To address the third research question, we examined whether the improvement in each knowledge domain differed significantly between the two groups. We calculated improvement scores for each student by subtracting their pretest score from their posttest score. These improvement scores were then compared between the two groups using the Mann-Whitney U test.

As shown in Table 9 and Figure 6, in conceptual knowledge the experimental group demonstrated greater median improvement of 3.250 (IQR = 2.250) compared to 1.750 (IQR = 3.500) for the control group. In procedural knowledge, the experimental group had a median improvement of 5.000 (IQR = 1.880), while the control group showed 4.250 (IQR = 3.750).

Table 9	Medians (and	IQR) for	Students In	nprovement b	y Group
---------	--------------	----------	-------------	--------------	---------

	Ν	Control	Experimental
Conceptual knowledge pretest	64	1.750 (3.500)	3.250 (2.250)
Procedural knowledge pretest	66	4.250 (3.750)	5.000 (1.880)

Mann–Whitney U tests confirmed that these differences were statistically significant (Table 10), with the experimental group outperforming the control group in both conceptual (U = 1469.000, p = .003, r = 0.304) and procedural knowledge (U = 1542.000, p = 0.008, r = 0.270).



 Table 10
 Mann-Whitney U Test for Improvement

	Statistic	р	Effect Size (Rank biserial correlation)
Conceptual knowledge pretest	1469.000	0.003	0.304
Procedural knowledge pretest	1542.000	0.008	0.270

These results indicate that the blended learning approach had a stronger impact on students' improvement, particularly in conceptual understanding. The findings confirm that the experimental group benefited more substantially from the intervention than the control group did from traditional instruction alone.

4 Discussion

The present study investigated the effectiveness of a blended learning intervention on students' conceptual and procedural knowledge of fractions, comparing it with traditional instruction. The findings offer valuable insights into the development of both domains through innovative, mobile-assisted educational practices.

Both the control and experimental groups demonstrated significant improvements from pretest to posttest in conceptual and procedural aspects of knowledge, as indicated by large effect sizes (rank biserial r ranging from 0.901 to 1.000). However, between-group comparisons of improvement scores revealed that the experimental group showed significantly greater gains in both domains compared to the control group. The median improvement in conceptual knowledge for the experimental group was almost double that of the control group (3.250 *vs.* 1.750), with a moderate effect size (r = 0.304). Similarly, procedural knowledge improvements were significantly higher in the experimental group (median 5.000 *vs.* 4.250; r = 0.270).

These results suggest that the blended instructional approach, which integrated digital, experiential, and collaborative learning tasks, was more effective in promoting deep understanding of fractional concepts than traditional methods. The significant procedural gains further support the effectiveness of the approach in promoting not only understanding but also the ability to apply fraction operations accurately.

The findings are particularly noteworthy in the context of fraction learning, a domain known to be challenging in mathematics education. The use of a variety of suitable representations, dynamic and collaborative exploration (see Figure 1 and 2), interactive activities, tangible materials (see Figure 3), student-made models and reflective activities supported deeper conceptual knowledge. These results are consistent with prior research highlighting the benefits of representational diversity (Arcavi, 2003; Wilkie & Roche, 2022; Zhang et al., 2014) and dynamic inquiry in mathematics education (Anat et al., 2020; Bulut et al., 2016; Nashiroh & Zainuddin, 2023; Palaigeorgiou et al., 2019; Poon, 2018; Reimer & Moyer-Packenham, 2005;

Thambi & Eu, 2013) and especially those with low initial performance (Moyer-Packenham et al., 2012).

Moreover, the digital tasks included in the intervention, particularly the interactive H5P activities and real-life problem solving tasks, fostered procedural fluency. Interactive formats enabled immediate feedback, repetitive practice and gradual scaffolding, which are key components in procedural learning (Rittle-Johnson & Schneider, 2015). This allowed students to practice procedures in meaningful contexts, rather than through isolated, rote exercises.

Multiple representations contributed to bridging the gap between conceptual and procedural knowledge. For example, dynamic models demonstrated the effect of multiplying fractions on quantities before students applied standard algorithms. This aligns with research suggesting that procedural fluency is best supported when built on a strong conceptual foundation (Rittle-Johnson & Schneider, 2015; Star et al., 2015). In the present study, the design of the intervention, with a structured progression from exploration to practice, encouraged this integration.

Another important aspect of the findings is that the experimental group showed balanced improvement in both conceptual and procedural fluency. In contrast, traditional instruction often favors procedural gains over deep conceptual understanding (Charalambous & Pitta-Pantazi, 2007; Lamon, 2007; Ni & Zhou, 2005; Siegler et al. 2013; Stafylidou & Vosniadou, 2004; Vam-vakoussi & Vosniadou, 2004). In this study, the blended learning group improved substantially and simultaneously in both domains. This indicates that the instructional design successfully integrated conceptual and procedural goals without overloading students or privileging one domain over the other. The use of tablets contributed to this balance by enabling exploration in dynamic environments that helped students visualize and manipulate fractions meaningfully. Additionally, hands-on activities and collaborative exploration encouraged students to reflect more deeply on fraction concepts while practicing operations.

In sum, the findings support the growing consensus that procedural knowledge is not isolated from conceptual knowledge, and that well-designed instructional environments can foster both domains in parallel (Byrnes & Wasik, 1991; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Schneider, 2015; Rittle-Johnson et al., 2001). These findings align with existing literature highlighting the effectiveness of the blended learning approach (Ampartzaki et al, 2024; Jailani et al., 2025; Laskaris et al, 2017; 2019; Motteram & Sharma, 2009), both in mathematics education (Ekeh et al., 2021; Fazal & Bryant, 2019; Iroko & Olaoye, 2021; Indrapangastuti et al., 2021; Kundu et al., 2021; Latif et al., 2024; Obot, 2023; Yaghmour, 2016) and general education (Cleveland-Innes & Wilton, 2018; Powell et. al., 2015).

Finally, the results contribute to broader discussions about the role of ICT and mobile learning in education (Goodwin, 2012; Green & Hannon, 2006; Prensky, 2001; Zaranis, 2014) and particularly in mathematics instruction (Arvanitaki & Zaranis, 2020; Attard & Curry, 2012; Clarke & Svanaes, 2014; Fabian et al., 2016; Fütterer et al., 2022).

4.1 Implications for Practice

The study demonstrates that mobile-supported blended instruction can be both feasible and highly effective in real classroom settings. Importantly, the tasks were delivered through widely available platforms, which means that the approach is accessible and scalable. This has practical implications for teachers and schools aiming to modernize and focus their mathematics instruction without requiring specialized hardware or software. The teaching material is created by the teacher himself/herself within open digital platforms (*e.g.* H5P, Geogebra) enabling tailor content to teaching objectives and students' needs.

Additional important implication is the concentration of all teaching resources in a single digital environment (such as a blog) simplifying organization, supporting continuity across activities and allowing students to revisit materials as needed.

Moreover, the intervention was designed to be aligned with the national curriculum, which facilitates its integration into standard teaching practice. Teachers can adopt this approach without sacrificing curriculum coverage, while providing students with more engaging and cognitively rich learning experiences.

Furthermore, the core elements of the blended intervention offer a flexible approach that can be adapted to other areas of mathematics, such as geometry or measurement, as well as subjects like science or language learning where conceptual understanding and active student participation are essential. The approach can also be modified for younger or older age groups by adjusting task complexity and scaffolds.

4.2 Limitations and Future Research

Although the findings of the present study are promising, several significant limitations should be acknowledged. First, the sample was limited to 130 5th-grade students from public primary schools in socioeconomically homogeneous areas of Heraklion, Crete. While pretest scores indicated baseline equivalence between groups, the limited demographic and geographic scope may restrict generalizability. Future studies should replicate this design across more diverse socioeconomic, cultural and educational contexts, as well as with different age groups, to assess broader applicability.

Second, the study focused on short-term learning gains without measuring long-term retention and transfer of fraction knowledge. Since procedural fluency often decays without ongoing reinforcement (Hecht & Vagi, 2010; Rittle-Johnson & Schneider, 2015), incorporating a delayed posttest or longitudinal follow-up would provide stronger evidence of the intervention's lasting impact.

Third, although implementation fidelity was monitored using structured weekly teacher checklists, variations in instructional delivery and practical challenges, such as frequent tablet charging and occasional Wi-Fi disconnections, could have affected the consistency of the blended delivery and students' engagement. Self-reported data, while informative, may not have fully captured these discrepancies.

Fourth, students' varying motivation levels were not controlled in this study, representing a limitation that future research should consider.

Fifth, this study did not include systematic qualitative data such as student reflections or detailed teacher observations. While informal teacher feedback suggested positive student engagement, the lack of formal qualitative insight limits a deeper understanding of participants' experiences and perceptions.

Future research should explore how this blended approach can be scaled or adapted for other mathematical domains or subjects as well as in varied classroom infrastructures. It would also be valuable to examine how different components of the intervention (*e.g.*, dynamic tasks *vs.* hands-on activities) contribute to learning outcomes, possibly through suitable experimental designs. Including qualitative measures in future studies may further enrich understanding of implementation processes and learner engagement. Overall, future studies should prioritize longitudinal assessment and rigorous control of implementation fidelity to build on the promising results reported here.

5 Conclusion

The present study explored the impact of a blended learning approach on fifth-grade students' in both conceptual and procedural knowledge. Through a structured comparison between a control group receiving traditional instruction and an experimental group engaging with digital and experiential activities, the study provides evidence that the mixed method significantly enhanced students' learning outcomes in both domains. These findings support the conclusion that integrating digital tools with hands-on, collaborative experiences offers added value in the teaching and learning of fractions.

Importantly, the success of this approach is rooted in both its content and format. Emphasizing the connection between fractional operations and their conceptual meaning emerged as a key strength, promoting deeper understanding rather than repetitive, sterile application. The results align with prior research emphasizing the role of multimodal learning in mathematical comprehension. By offering students multiple pathways to engage with mathematical concepts, the blended learning approach proved especially effective in bridging the often-observed gap between knowing how to perform operations and understanding the principles behind them.

A key aspect of the intervention was the use of tablets, which allowed students to interact with dynamic tasks in a direct, tactile way. The touchscreen interface supported active manipulation of visual representations, fostering stronger engagement and deeper cognitive connections. Furthermore, integrating mobile-friendly, open-access platforms into primary mathematics education represents a novel and scalable pedagogical approach enabling educators to design personalised, curriculum-aligned content that transcends the specific limitations of educational apps.

Additionally, the adaptability of this approach to other math domains and educational contexts remains a promising avenue. Future research might focus on how such blended designs influence

long-term understanding and can be tailored across age groups and subjects, helping to refine sustainable models of technology-integrated instruction.

Conflicts of interest

The authors declare that they have no conflict of interest.

References

- Abdul Latif, N. H., Shahrill, M., & Hidayat, W. (2024). Mastering fractions and innovating with the station rotation model in blended learning. Infinity Journal, 13(2), 501–530. https://doi.org/10.22460/infinity.v13i2.p501-530
- Ampartzaki, M., Tassis, K., Kalogiannakis, M., Pavlidou, V., Christidis, K., Chatzoglidou, S., & Eleftherakis, G. (2024). Assessing the Initial Outcomes of a Blended Learning Course for Teachers Facilitating Astronomy Activities for Young Children. Education Sciences, 14(6), 606. https://doi.org/10.3390/educsci14060606
- Anat, K., Shirley, R., & Hanna, L.-Z. (2019). Building a computerized dynamic representation as an instrument for mathematical explanation of division of fractions. International Journal of Mathematical Education in Science and Technology, 51(2), 247–264. https://doi.org/10.1080/0020739x.2019.1648888
- Arcavi, A. (2003). Educational Studies in Mathematics, 52(3), 215–241. https://doi.org/10.1023/a:1024312321077
- Arvanitaki, M., & Zaranis, N. (2020). The use of ICT in teaching geometry in primary school. Education and Information Technologies, 25(6), 5003–5016. https://doi.org/10.1007/s10639-020-10210-7
- Attard, C., & Curry, C. (2012). Exploring the use of iPads to engage young students with mathematics. In J. Dindyal, L. P. Cheng, & S. F. Ng (Eds.), Mathematics education: expanding horizons. Proceedings of the 35th Annual Conference of the Mathematics Education Research Group of Australasia. (MERGA-35) (pp. 75-82). MERGA.
- Attard, C., & Orlando, J. (2014). Early career teachers, mathematics and technology: device conflict and emerging mathematical knowledge. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), Curriculum in focus: Research guided practice. Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia. (MERGA-37) (pp. 71-78). MERGA.
- Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number. Ratio and proportion. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning. MacMillan.
- Behr, M., Lesh, R., Post, T., & Silver E. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), Acquisition of Mathematics Concepts and Processes, (pp. 91-125). Academic Press.
- Bulut, M., Akçakın, H. Ü., Kaya, G., & Akçakın, V. (2016). The Effects of GeoGebra On Third Grade Primary Students' Academic Achievement in Fractions. International Electronic Journal of Mathematics Education, 11(2), 347–255. https://doi.org/10.29333/iejme/338
- Byrnes, J. P., & Wasik, B. A. (1991). Role of conceptual knowledge in mathematical procedural learning. Developmental Psychology, 27(5), 777–786. https://doi.org/10.1037/0012-1649.27.5.777
- Charalambous, C. Y., & Pitta-Pantazi, D. (2006). Drawing on a Theoretical Model to Study Students' Understandings of Fractions. Educational Studies in Mathematics, 64(3), 293–316. https://doi.org/10.1007/s10649-006-9036-2
- Chilivumbo, C. (2015). Mobile e-learning: The choice between Responsive/Mobile Websites and Mobile Applications for Virtual Learning Environments for increasing access to Higher Education in Malawi. 2015 IST-Africa Conference, 1–15. https://doi.org/10.1109/istafrica.2015.7190520
- Clarke, B., & Svanaes, S. (2014). An updated literature review on the use of tablets in education. Family kids and youth, 1-20.

https://www.kidsandyouth.com

Cleveland-Innes, M., & Wilton, D. (2018). Guide to Blended Learning. https://doi.org/10.56059/11599/3095

- Cohen, L., Manion, L., & Morrison, K. (2017). Research Methods in Education. Routledge. https://doi.org/10.4324/9781315456539
- Creswell, J. (2016) Research in Education: Design, conduct and evaluation of quantitative and qualitative research (Translated by Kouvarakou, N.). Ion. (Original work published 2012).
- Dogan, A., & IŞIK TERTEMİZ, N. (2020). Fraction models used by primary school teachers. İlköğretim Online, 1888–1901.

https://doi.org/10.17051/ilkonline.2020.762538

- Ekeh, M. C., Venketsamy, R., Unamba, E. C., & Ugochukwu, N. J. (2021). Using blended-learning strategy in mathematics to strengthen the teaching of geometric 2D shapes and their properties to primary 6 learners. The Independent Journal of Teaching and Learning, 16(2). https://hdl.handle.net
- Escuder, A., & Furner, J. M. (2011). The impact of GeoGebra in math teachers' professional development. In Proceedings of the International Conference on Technologies in Collegiate Mathematics (pp. 76-84). Pearson.
- Fabian, K., Topping, K. J., & Barron, I. G. (2015). Mobile technology and mathematics: effects on students' attitudes, engagement, and achievement. Journal of Computers in Education, 3(1), 77–104. https://doi.org/10.1007/s40692-015-0048-8
- Fazal, M., & Bryant, M. M. (2019). Blended learning in middle school math: The question of effectiveness. Journal of Online Learning Research, 5(1), 49-64.
- Fütterer, T., Scheiter, K., Cheng, X., & Stürmer, K. (2022). Quality beats frequency? Investigating students' effort in learning when introducing technology in classrooms. Contemporary Educational Psychology, 69, 102042.

https://doi.org/10.1016/j.cedpsych.2022.102042

Gazzawe, F., Mayouf, M., Lock, R., & Alturki, R. (2022). The Role of Machine Learning in E-Learning Using the Web and AI-Enabled Mobile Applications. Mobile Information Systems, 2022, 1–10. https://doi.org/10.1155/2022/3696140

Goodwin, K. (2012). Use of Tablet Technology in the Classroom. NSW Curriculum and Learning Innovation Centre.

Goodwin, K., & Highfield, K. (2012). iTouch and iLearn: An examination of "educational" apps. In Early education and technology for children conference (pp. 14-16). Salt Lake City, Utah, USA.

Green, H., & Hannon, C. (2006). Their Space. Education for a digital generation. DEMOS.

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. Journal of Educational Psychology, 102(4), 843–859.

https://doi.org/10.1037/a0019824

Hiebert, J. (Ed.). (2013). Conceptual and Procedural Knowledge. Routledge. https://doi.org/10.4324/9780203063538

Indrapangastuti, D., Surjono, H. D., S., & Yanto, B. E. (2021). Effectiveness of the Blended Learning Model to Improve Students' Achievement of Mathematical Concepts. Journal of Education and E-Learning Research, 8(4), 423–430.

https://doi.org/10.20448/journal.509.2021.84.423.430

- Iroko, G. A., & Olaoye, A. A. (2021). Potency of Blended Strategy on Students' Performance in Algebra at Senior Secondary Schools. International Journal of Innovative Research and Development, 10(3). https://doi.org/10.24940/ijird/2021/v10/i3/mar21039
- Jailani, N., Rosli, R., & Mahmud, M. S. (2025). Factors Influencing Mathematics Teachers' Blended Learning: A Systematic Review. International Journal of Learning, Teaching and Educational Research, 24(1), 397–419.

https://doi.org/10.26803/ijlter.24.1.20

- Kalogiannakis, M., & Papadakis, S. (2020). The Use of Developmentally Mobile Applications for Preparing Pre-Service Teachers to Promote STEM Activities in Preschool Classrooms. Mobile Learning Applications in Early Childhood Education, 82–100. https://doi.org/10.4018/978-1-7998-1486-3.ch005
- Kalra, P. B., Hubbard, E. M., & Matthews, P. G. (2020). Taking the relational structure of fractions seriously: Relational reasoning predicts fraction knowledge in elementary school children. Contemporary Educational Psychology, 62, 101896.

https://doi.org/10.1016/j.cedpsych.2020.101896

- Kieren, T. E. (1976). On the mathematical, cognitive, and instructional foundations of rational numbers. In R. Lesh (Ed.), Number and measurement: Papers from a research workshop (pp. 101-144). ERIC/SMEAC.
- Kieren, T. E. (1980). The rational number construct-its elements and mechanisms. In T. E. Kieren (Ed.), Recent research on number learning (pp. 125-150). Columus, ERIC/SMEAC.
- Kieren, T. E. (1993). Rational and fractional numbers: from quotient fields to recursive understanding. In T. Carpenter, E. Fennema & T. Romberg (Eds.), Rational numbers: An integration of research (pp. 49-84). Lawrence Erlbaum.
- Koleza, E. (2000). Epistemological and didactical approach to elementary mathematical concepts [in Greek]. Leader Books.
- Kundu, A., Bej, T., & Rice, M. (2020). Time to engage: Implementing math and literacy blended learning routines in an Indian elementary classroom. Education and Information Technologies, 26(1), 1201–1220.

https://doi.org/10.1007/s10639-020-10306-0

Lamon, S. J. (2020). Teaching Fractions and Ratios for Understanding. Routledge. https://doi.org/10.4324/9781003008057

- Lamon, S.J. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp. 629-667). NCTM.
- Laskaris, D., Heretakis, E., Kalogiannakis, M., & Ampartzaki, M. (2019). Critical reflections on introducing e-learning within a blended education context. International Journal of Technology Enhanced Learning, 11(4), 413. https://doi.org/10.1504/ijtel.2019.102550

Laskaris, D., Kalogiannakis, M., & Heretakis, E. (2017). "Interactive evaluation" of an e-learning course within the context of blended education. International Journal of Technology Enhanced Learning, 9(4), 339.

https://doi.org/10.1504/ijtel.2017.087793

Mayer, R. E. (2009). Multimedia learning (2nd ed.). Cambridge University Press.

Mayer, R. E. (2014). Cognitive Theory of Multimedia Learning. The Cambridge Handbook of Multimedia Learning, 43–71.

https://doi.org/10.1017/cbo9781139547369.005

- Moss, J. (2005). Pipes, tubes, and beakers: New approaches to teaching the rational-number system. In M. S. Donovan & J. D. Bransford (Eds.), How students learn: Mathematics in the classroom (pp. 121-162). National Academies Press.
- Motteram, G., & Sharma, P. (2009). Blending learning in a web 2.0 world. International Journal of Emerging Technologies and Society, 7(2), 83-112. https://www.academia.edu
- Moyer-Packenham, P. S., Ulmer, L. A., Anderson, K. L. (2012). Examining pictorial models and virtual manipulatives for third-grade fraction instruction. Interactive Online Learning. 11(3), 103-120.
- Nashiroh, F., & Zainuddin, A. (2023). Development of GeoGebra-Based Fraction Gap Learning Media to Improve Understanding of the Fraction Concept of Grade V Elementary School. DIDAKTIKA TAUHIDI: Jurnal Pendidikan Guru Sekolah Dasar, 10(1), 55–69. https://doi.org/10.30997/dt.v10i1.8238
- Ni, Y., & Zhou, Y.-D. (2005). Teaching and Learning Fraction and Rational Numbers: The Origins and Implications of Whole Number Bias. Educational Psychologist, 40(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3
- Niemi, D. (1996). Instructional influence on content area explanations and representational knowledge: Evidence for the construct validity of measures of principled understanding. (CSE Technical Report 403). National Center for Research on Evaluation, Standards, and Student Testing. University of California.

https://cresst.org

Obot, P. F. (2023). Impact of blended learning approach on students' achievement in quadratic and simultaneous equations. International Journal of Trend in Scientific Research and Development, 7(6), 552-558.

https://www.ijtsrd.com

- Palaigeorgiou, G., Tsolopani, X., Liakou, S., & Lemonidis, C. (2019). Movable, Resizable and Dynamic Number Lines for Fraction Learning in a Mixed Reality Environment. The Challenges of the Digital Transformation in Education, 118–129. https://doi.org/10.1007/978-3-030-11935-5_12
- Papadakis, S. (2020). Apps to Promote Computational Thinking Concepts and Coding Skills in Children of Preschool and Pre-Primary School Age. Mobile Learning Applications in Early Childhood Education, 101–121.

https://doi.org/10.4018/978-1-7998-1486-3.ch006

- Papadakis, S., Alexandraki, F., & Zaranis, N. (2021). Mobile device use among preschool-aged children in Greece. Education and Information Technologies, 27(2), 2717–2750. https://doi.org/10.1007/s10639-021-10718-6
- Papadakis, S., Kalogiannakis, M., & Zaranis, N. (2021). Teaching mathematics with mobile devices and the Realistic Mathematical Education (RME) approach in kindergarten. Advances in Mobile Learning Educational Research, 1(1), 5–18. https://doi.org/10.25082/amler.2021.01.002
- Papadakis, S., Trampas, A., Barianos, A., Kalogiannakis, M., & Vidakis, N. (2020). Evaluating the Learning Process: The "ThimelEdu" Educational Game Case Study. Proceedings of the 12th International Conference on Computer Supported Education. https://doi.org/10.5220/0009379902900298
- Poon, K. K. (2017). Learning fraction comparison by using a dynamic mathematics software GeoGebra. International Journal of Mathematical Education in Science and Technology, 49(3), 469–479. https://doi.org/10.1080/0020739x.2017.1404649
- Post, T., Cramer, K., Behr, M., Lesh, R., & Harel, G. (1993). Curriculum implications of research on the learning, teaching and assessing of rational number concepts. In T. Carpenter, E. Fennema & T. Romberg (Eds.), Rational numbers: An integration of research (pp. 327-361). Lawrence Erlbaum.
- Powell, A., Watson, J., Staley, P., Patrick, S., Horn, M., Fetzer, L., Hibbard, L., Oglesby, J., & Verma, S. (2015). Blending learning: The evolution of online and face-to-face education from 2008-2015. Promising Practices in Blended and Online Learning Series. https://files.eric.ed.gov
- Prensky, M. (2001). Digital Natives, Digital Immigrants Part 1. On the Horizon, 9(5), 1–6. https://doi.org/10.1108/10748120110424816
- Reimer, K., & Moyer-Packenham, P. S. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. Journal of Computers in Mathematics and Science Teaching, 24(1), 5-25.
- Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? Journal of Educational Psychology, 91(1), 175–189. https://doi.org/10.1037/0022-0663.91.1.175

- Rittle-Johnson, B., & Schneider, M. (2014). Developing Conceptual and Procedural Knowledge of Mathematics. The Oxford Handbook of Numerical Cognition, 1118–1134. https://doi.org/10.1093/oxfordhb/9780199642342.013.014
- Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. Journal of Educational Psychology, 93(2), 346–362.

https://doi.org/10.1037/0022-0663.93.2.346

- Schneider, M., & Siegler, R. S. (2010). Representations of the magnitudes of fractions. Journal of Experimental Psychology: Human Perception and Performance, 36(5), 1227–1238. https://doi.org/10.1037/a0018170
- Siegler, R. S., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. Journal of Educational Psychology, 107(3), 909–918. https://doi.org/10.1037/edu0000025
- Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: the new frontier for theories of numerical development. Trends in Cognitive Sciences, 17(1), 13–19. https://doi.org/10.1016/j.tics.2012.11.004
- Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. Learning and Instruction, 14(5), 503–518. https://doi.org/10.1016/j.learninstruc.2004.06.015
- Staker, H., & Horn, M.B. (2012). Classifying K-12 blended learning. Innosight Institute. https://www.christenseninstitute.org
- Star, J. R. (2005). Reconceptualizing procedural knowledge. Journal for Research in Mathematics Education, 36(5), 404-411.

https://doi.org/10.2307/30034943

- Thambi, N., & Eu, L. K. (2013). Effect of students' achievement in fractions using GeoGebra. SAINSAB, 16, 97-106.
- UNESCO. (2013). Policy guidelines for mobile learning. UNESCO.

http://unesdoc.unesco.org

Vamvakoussi, X., & Vosniadou, S. (2012). Bridging the Gap Between the Dense and the Discrete: The Number Line and the "Rubber Line" Bridging Analogy. Mathematical Thinking and Learning, 14(4), 265–284.

https://doi.org/10.1080/10986065.2012.717378

- Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2016). Elementary and middle school mathematics: Teaching developmentally (9th Ed.). Pearson Education.
- Wilkie, K. J., & Roche, A. (2022). Primary teachers' preferred fraction models and manipulatives for solving fraction tasks and for teaching. Journal of Mathematics Teacher Education, 26(6), 703–733. https://doi.org/10.1007/s10857-022-09542-7
- Yaghmour, K. S. (2016). Effectiveness of blended teaching strategy on the achievement of third grade students in mathematics. Journal of Education and Practice, 7(5), 65-73.
- Zaranis, N. (2014). The teaching of subtraction using ICT in kindergarten. In Proceedings of the 10th International Symposium EUTIC 2014 (pp. 563-574), Lisbon, Portugal.
- Zhang, X., Clements, M. A., & Ellerton, N. F. (2014). Conceptual mis(understandings) of fractions: From area models to multiple embodiments. Mathematics Education Research Journal, 27(2), 233–261. https://doi.org/10.1007/s13394-014-0133-8