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RESEARCH ARTICLE

Dynamic Visualization and Conceptual Understanding of Circle Theorems Through Teacher-Mediated GeoGebra Use: An Action Research in a Grade 10 Classroom in Nepal

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Abstract: Developing conceptual understanding in geometry remains challenging when circle theorems are learned primarily through memorization, static diagrams, and procedural exercises. Dynamic geometry environments may provide opportunities for students to explore geometric relationships, but how such exploration contributes to mathematical reasoning requires closer qualitative examination. This study explored how Grade 10 students developed conceptual understanding of circle theorems during GeoGebra-supported learning activities in a secondary mathematics classroom in Pokhara, Nepal. A qualitative action-research design was conducted over 15 instructional days with 32 students, with six focal participants selected for detailed analysis. The intervention comprised three cycles of planning, action, observation, and reflection. GeoGebra applets were projected using a single teacher-operated laptop, with limited shared student manipulation during selected group activities. Data were generated through classroom observations, student worksheets and reflective responses, semi-structured interviews, and a reflective journal, and were analyzed thematically. Three interconnected themes emerged: developing conceptual understanding through dynamic visualization; moving from dynamic observation to mathematical reasoning and justification; and peer interaction and teacher scaffolding in mathematical reasoning. Students increasingly recognized invariants, explored conjectures, and connected visual observations with mathematical explanations, although some initially relied on visual appearance or repeated measurements without sufficient justification. The findings position GeoGebra as a teacher-mediated resource rather than an independent cause of conceptual learning. Its educational value depended on purposeful task design, teacher questioning, peer interaction, and opportunities to connect dynamic exploration with mathematical reasoning, particularly in resource-constrained classrooms.

Keywords: GeoGebra, teacher-mediated technology, dynamic visualization, invariance, Nepal

1 Introduction

Numerous studies have shown that when the focus is on the lecturer using static illustrations in teaching geometry, it may not enable learners to have a deeper understanding of the subject (Hollebrands et al., 2023; Jones, 2003). Most mathematics lessons at the secondary level are still focused on exam preparation rather than concept development. This implies that while students will be able to complete tasks from the textbooks, it will not be easy for them to apply what they have learned about geometry into different contexts (National Council of Teachers of Mathematics, 2014). The issue is particularly evident in circle theorem learning, where dynamic visualization of geometrical characteristics is essential. GeoGebra offers a combined application for constructions, graphing, and dynamic visualization (Hohenwarter & Jones, 2007). Results of the research reveal that the use of dynamic visualization software assists students in communicating actively and mathematically, which contributes to their understanding of concepts through constructing and manipulating objects to discover invariants (Boonen et al., 2014). The visualization afforded by GeoGebra can help learners connect dynamic visual representations with geometric properties and reasoning, supporting their understanding of otherwise abstract geometric concepts (Dahal et al., 2022). It has previously been observed that the integration of GeoGebra in the field of math education enhances geometry theorems understanding, geometrical visualization, and problem-solving abilities (Chimuka, 2017; Zengin et al., 2012). Besides, previous literature has established that working on tasks together with dynamic visualization results in math communication and

reflection (Bakar et al., 2021). While the benefits offered by dynamic visualizations in mathematics have been identified extensively in previous studies, most research conducted so far has been primarily focused on academic achievement and students' attitudes toward the use of technologies.

1.1 Background and Problem Context

The present study is particularly relevant in developing contexts such as Nepal, where opportunities for technology-enhanced mathematics learning remain limited despite policy-level encouragement of ICT integration. Although the incorporation of ICT tools in the teaching process has been recommended by government education policies, there have been various challenges to making effective use of ICT in mathematics instruction (Ministry of Education, 2013). According to Thapa et al. (2022), in the process of conducting a teaching experiment for learning the circle concept, it was noted that GeoGebra was learner-focused rather than teacher-focused because GeoGebra facilitates the process of understanding mathematical concepts through visualization and collaboration. Likewise, according to Pant et al. (2024), mathematics teachers considered GeoGebra as an effective tool that helped students relate abstract and concrete features of mathematics with visualization. In addition, this point can also be applied to the results obtained by Lamichhane et al. (2023), in which they noted that the use of GeoGebra in teaching trigonometry enhanced conceptual thinking, visualization, classroom engagement, and mathematical communication skills of students. Thus, it is clear that from the above-mentioned research studies, there is enough empirical evidence for the role of GeoGebra in constructive mathematics learning.

At present, the teaching of geometry in secondary education institutions is largely teacher-centered. This is because, in most cases, it involves the use of textbooks and diagrams. It becomes a common problem for students to comprehend abstract geometrical concepts through visual representations because of an inadequate use of ICT in their instruction. This study was informed by Vygotsky's Social Constructivist Theory, which views learning as a socially mediated process developed through interaction, collaboration, and guided participation (Vygotsky, 1978). The main idea behind this particular theoretical framework is that learning is a social phenomenon that takes place because of interaction, collaboration, and guided participation. Within this perspective, GeoGebra functions as a mediational tool that supports learners' cognitive development by facilitating exploration, communication, and collaborative knowledge construction.

Digital technology has increasingly become part of mathematics classrooms, not simply as a means of presenting mathematical information, but as a resource through which students can explore, represent, and reason about mathematical ideas. In geometry, this potential is particularly important because many relationships are difficult to appreciate from static diagrams alone. Dynamic mathematics environments allow learners to construct figures, manipulate their elements, observe what changes and what remains unchanged, and use these observations to develop mathematical conjectures. The National Council of Teachers of Mathematics (National Council of Teachers of Mathematics, 2014) has therefore emphasized the appropriate use of technology to support meaningful mathematical exploration, reasoning, and problem solving.

1.2 Dynamic Visualization and GeoGebra in Mathematics Learning

Among the available digital tools, GeoGebra has received considerable attention in mathematics education because it provides a dynamic environment in which geometric objects can be constructed and manipulated while their mathematical relationships remain visible. Developed initially by Markus Hohenwarter, GeoGebra integrates dynamic geometry with algebraic and other mathematical representations. In a geometry lesson, for example, students can drag a point on a circle and immediately observe corresponding changes in angles, chords, or arcs. Such interaction creates opportunities for students to notice patterns, test conjectures, and discuss why particular relationships remain invariant. However, the educational value of such technology does not lie in visualization alone. What students notice through a dynamic representation still needs to be connected with mathematical explanation and justification.

This issue is particularly relevant to the teaching and learning of circle theorems. Understanding circle theorems requires students to coordinate several related concepts, including arcs, chords, central angles, inscribed angles, and cyclic quadrilaterals. In conventional classroom settings, these relationships are often introduced through static diagrams, theorem statements, worked examples, and repeated practice. Students may therefore learn to recall a theorem and apply an associated procedure without necessarily developing a clear understanding of why the relationship holds. Research in the Nepalese secondary-school context has shown that students can experience difficulties in understanding abstract geometric concepts, while dynamic visualization through GeoGebra can provide opportunities to explore and make sense of geometric relationships (Thapa

et al., 2022). Thus, the challenge is not simply whether students can obtain a correct numerical answer, but whether they can recognize a relationship across different configurations, explain it mathematically, and eventually justify its validity.

The problem is also meaningful in the context of secondary mathematics education in Nepal. Although the secondary mathematics curriculum places importance on conceptual understanding, problem solving, and mathematical reasoning, classroom instruction can remain strongly oriented toward examination preparation, procedural practice, and teacher-led explanation. In such settings, students may have limited opportunities to manipulate geometric objects and investigate relationships for themselves. During my classroom experience, I observed that some Grade 10 students could remember statements of circle theorems but experienced difficulty explaining the relationships among the geometric elements represented in a diagram. This provided an important practical reason for exploring whether a dynamic geometry environment could create opportunities for students to develop a deeper understanding of these relationships.

Although previous studies have reported positive contributions of GeoGebra to students' achievement, visualization, engagement, and mathematical communication, less attention has been given to how students actually develop conceptual understanding while working with dynamic representations of circle theorems. In particular, there is limited classroom-based evidence showing how students move from observing changing geometric configurations to recognizing invariants, forming conjectures, explaining relationships, and connecting these observations with formal mathematical justification. This issue is particularly relevant in the Nepalese secondary mathematics context, where geometry is often taught through textbook diagrams and procedural exercises and where access to digital resources may be limited. Therefore, this study examined how Grade 10 students developed conceptual understanding of circle theorems through GeoGebra-supported learning activities. Rather than treating GeoGebra simply as a tool for improving achievement, the study focused on the learning processes that occurred as students explored, discussed, questioned, and explained geometric relationships.

2 Literature Review

2.1 Technology-supported Mathematics Learning

Technology-supported mathematics learning has increasingly shifted attention from presenting mathematical information toward creating opportunities for students to explore, represent, and communicate mathematical ideas. Research on digital mathematics environments suggests that dynamic representations can help students examine relationships, test conjectures, and connect visual and symbolic forms of mathematical thinking (Hollebrands et al., 2023). Such environments may be particularly valuable in geometry, where students are required to coordinate diagrams, spatial relationships, measurements, and deductive reasoning.

At the same time, the literature suggests that technology does not automatically produce conceptual understanding. Its value depends on the mathematical tasks, classroom interaction, and teacher guidance through which students interpret what they see. Thus, technology is better understood as a resource that can reshape opportunities for mathematical activity rather than as an instructional solution in itself.

2.2 GeoGebra and Geometry Learning

Among digital mathematics environments, GeoGebra has received considerable attention because it allows learners to construct, manipulate, and examine mathematical objects within a connected visual environment (Hohenwarter & Jones, 2007). Previous studies have reported that GeoGebra can support students' geometric visualization, conceptual understanding, problem solving, and mathematical communication (Zengin et al., 2012; Chimuka, 2017; Bakar et al., 2021). More recent research has also emphasized the value of dynamic representations for helping students identify invariants and connect visual observations with mathematical reasoning (Boonen et al., 2014). Studies conducted in Nepal similarly suggest that GeoGebra can support learner-centered exploration and help students connect abstract mathematical ideas with visual representations (Thapa et al., 2022; Lamichhane et al., 2023; Pant et al., 2024).

However, much of this literature reports outcomes such as achievement, attitudes, visualization, or general engagement. Comparatively less attention has been given to what happens during the learning process: how students interpret dynamic representations, how their initial observations develop into mathematical explanations, and how teacher questioning and peer interaction shape this movement. This distinction is important for circle theorems, where seeing that a relationship

remains unchanged is only an initial step toward understanding why the relationship holds.

2.3 Challenges in Dynamic Geometry Learning

Dynamic geometry environments also present important challenges. Students may focus on dragging objects, observing measurements, or confirming that a relationship appears unchanged without developing an explanation for why it holds. Dynamic visualization can therefore support conjecture formation while still leaving students dependent on empirical evidence rather than deductive reasoning. This issue is particularly relevant in circle geometry, where visual configurations can appear convincing even when students have not yet understood the underlying theorem. Previous research has emphasized the importance of teacher questioning, task design, and mathematical discussion in helping learners move from visual exploration toward explanation and justification (Boonen et al., 2014). In other words, the difficulty is not simply whether students can use dynamic software, but whether they can use what they see as a basis for mathematical reasoning. This challenge is especially relevant in the Nepalese secondary-school context, where mathematics learning remains strongly influenced by examination demands and access to digital learning environments is uneven (Ministry of Education, 2013). Although existing Nepalese studies have reported positive experiences with GeoGebra, there remains limited classroom-based evidence showing how students actually make sense of dynamic representations while learning circle theorems, including the difficulties they encounter and the support they require. The present study therefore focuses not simply on whether GeoGebra is useful, but on how students' conceptual understanding develops through GeoGebra-supported classroom activity and interaction.

3 Theoretical Perspective

This study draws primarily on Vygotsky's sociocultural theory, which views learning as a process mediated by cultural tools and developed through interaction with others (Vygotsky, 1978). This perspective is particularly relevant because students did not learn circle theorems through GeoGebra in isolation. They interacted with dynamic representations, responded to teacher questions, discussed observations with peers, and gradually connected what they saw with mathematical explanations. In this process, GeoGebra functioned as a mediating resource through which students could manipulate geometric constructions, observe changing relationships, test ideas, and communicate their mathematical thinking. The notion of the Zone of Proximal Development (ZPD) is also useful for interpreting how teacher questioning and peer interaction supported students when they could recognize a relationship but could not yet explain it independently. Through prompts such as *"What remains unchanged?" "Why does this relationship remain true?"*, and *"How can you justify your answer?"*, students were encouraged to move beyond visual observation toward mathematical explanation. Peer discussion similarly provided opportunities for students to compare interpretations and refine their reasoning. These interactions can be understood as part of the mediation through which students gradually internalized mathematical relationships that were initially explored through dynamic representations and social interaction. A cognitive learning perspective also informed the interpretation of students' conceptual development. From this perspective, learning involves actively connecting new experiences with existing knowledge rather than simply memorizing isolated rules. In this study, dynamic manipulation allowed students to examine geometric relationships across multiple configurations and gradually connect visual observations with mathematical ideas. Thus, the theoretical perspectives together enabled the study to interpret students' learning as both an active process of constructing mathematical understanding and a socially mediated process supported by tools, teacher scaffolding, and peer interaction.

4 Research Gap, Purpose, and Research Question

Existing research provides substantial evidence that GeoGebra and other dynamic mathematics environments can support students' visualization, engagement, achievement, and mathematical communication. However, comparatively less attention has been given to the process through which students develop conceptual understanding while working with dynamic representations. In particular, limited qualitative evidence explains how secondary students move from observing geometric relationships to recognizing invariants, forming conjectures, explaining their reasoning, and connecting empirical observations with mathematical justification. This gap is particularly meaningful in the context of Nepal, where secondary geometry teaching often takes place within examination-oriented classrooms, and access to digital resources is not always extensive. A classroom-based investigation can therefore provide a more situated understanding of what GeoGebra makes possible and what remains dependent on teacher guidance, peer interaction, and instructional design. Against this background, the present study explored students' developing conceptual understanding

of circle theorems during a 15-day GeoGebra-supported action-research intervention. The study was guided by the following research question: *How did Grade 10 students develop conceptual understanding of circle theorems during GeoGebra-supported learning activities?*

5 Methods

This study adopted an action-research approach to explore how GeoGebra-supported dynamic visualization was experienced by Grade 10 students while learning circle related theorems. Action research was considered appropriate because it allowed me, as the teacher-researcher, to examine students' learning within the natural classroom setting, reflect on what was happening during the lessons, and make instructional adjustments based on students' emerging needs (Kemmis et al., 2014). Rather than treating GeoGebra as an isolated intervention, the study examined its use together with teacher questioning, student exploration, discussion, and collaborative learning. The study followed the action-research process of planning, acting, observing, and reflecting.

At the beginning, students' difficulties with circle-theorem concepts were identified through classroom observations, discussions, and a diagnostic pre-test. Based on these initial observations, GeoGebra-supported activities were planned to provide students with opportunities to explore geometric relationships dynamically. The intervention was carried out over 15 instructional days and organized into three action-research cycles. During each cycle, students' responses, mathematical discussions, written work, and interactions with the GeoGebra activities were observed and documented. After each cycle, I reflected on students' difficulties, misconceptions, and ways of reasoning and used these reflections to adjust the activities and teacher questioning in the following cycle. In this way, the research process moved from planning to action, observation, reflection, and further instructional adjustment. The focus was therefore not simply on whether GeoGebra was effective, but on how students' mathematical reasoning developed as they explored, discussed, tested, and explained circle-theorem relationships within a GeoGebra-supported classroom environment.

5.1 Action Research as Research Design

The intervention was carried out over 15 instructional days and was organized into three sequential action-research cycles. In each cycle, I planned the lessons, implemented the activities, observed students' responses, and reflected on the classroom experiences. The two co-authors were not present in the classroom and joined the study after data generation; "we" is therefore used in this article only for work completed jointly by all three authors. The first cycle mainly focused on helping students become familiar with GeoGebra and explore circle-theorem relationships through dynamic visualization. Based on the difficulties and responses observed in this cycle, the teaching activities were adjusted in the second cycle to give more attention to conjecture formation, explanation, and peer discussion. The third cycle then focused more on connecting students' observations with mathematical reasoning, problem-solving, and formal justification. In this way, reflection was used continuously to improve the teaching activities in the following cycle.

5.2 Research Context and Participants

The study was conducted in a secondary school in Pokhara Valley, Nepal, with a Grade 10 class studying compulsory mathematics. A total of 32 students participated in the 15-day instructional intervention. The whole class was involved in the lessons, classroom activities, diagnostic pre-test, post-test, student work, and classroom observations. For more detailed qualitative exploration, six students were selected as focal participants: Manish, Nishu, Prayag, Bibek, Grishma, and Yesh. All participants were between 16 and 18 years of age, however, because it was not practical to conduct detailed interviews and follow the learning experiences of all 32 students throughout the intervention, six students were selected as focal participants for in-depth qualitative analysis. The achievement levels were based on students' previous terminal examination marks, obtained before the intervention, and were used to identify participants with relatively high, moderate, and relatively lower initial achievement. All six focal participants and their parents or guardians provided consent for participation in the interviews and for the reproduction of their written work in this study. Table 1 illustrates the profile of the participants.

Table 1 illustrates the basis for selecting the six focal participants and shows that the qualitative analysis was not limited to students who were already high achieving or highly vocal. Their different levels and patterns of participation allowed us to examine how students with varied classroom experiences responded to and made sense of the GeoGebra-supported activities. The six focal students were selected purposively to capture variation in students' classroom participation, mathematical performance, and willingness to articulate their learning experiences. Rather than

Table 1 Profile of the Six Focal Participants

Pseudonym	Gender	Prior Mathematics Achievement	Classroom Participation	Reason for Inclusion
Manish	Male	Relatively high	Active	To represent a student who demonstrated comparatively strong achievement and regular participation
Nishu	Female	Moderate	Active	To represent a student with moderate achievement who participated consistently in classroom activities
Prayag	Male	Relatively high	Moderate	To represent a student who demonstrated good mathematical performance but participated selectively in discussions
Bibek	Male	Moderate	Moderate	To represent a student with developing understanding and moderate classroom participation
Grishma	Female	Moderate	Increasing	To represent a student whose participation became more active during the intervention
Yesh	Male	Relatively lower initially	Initially limited, later increasing	To represent a student who was initially less vocal but became increasingly involved in exploration and discussion

selecting students solely on the basis of high or low achievement, we considered multiple sources of classroom information, including students' participation during mathematics lessons, responses to classroom questions and activities, performance in regular mathematics work, and their willingness to participate in interviews and reflect on their learning. This approach allowed us to include students with different patterns of participation and mathematical engagement rather than relying on a single measure of achievement. Selection was made before the detailed qualitative analysis began, and the same criteria were applied across the class as far as possible. The six students were treated as focal cases for closer examination, not as statistically representative of the entire class. Their experiences were interpreted alongside whole-class observations, student work, and other sources of evidence. This helped us examine how students with different levels of participation and mathematical engagement experienced the GeoGebra-supported activities while avoiding the assumption that the experiences of six students represented all 32 students.

The instructional focus was the Grade 10 topic of Circle Theorems. This topic was selected because it involves relationships among angles, arcs, chords, central angles, inscribed angles, and cyclic quadrilaterals, which can be difficult for students to understand through static diagrams and procedural explanations alone. Based on my previous classroom experience with this topic, I observed that students often had difficulty connecting the visual features of geometric figures with the underlying mathematical relationships. This provided the practical basis for introducing dynamic GeoGebra activities to support students' exploration and discussion of circle theorems.

5.3 Researcher Roles and Reflexivity

As the classroom teacher and teacher-researcher, I was closely involved in designing and implementing the intervention, collecting data, conducting interviews, and interpreting students' learning experiences. I recognized that this dual role could influence classroom interactions, students' responses, and my interpretation of the data. To remain critically aware of this influence, I maintained a reflective journal throughout the study, recording classroom events, instructional decisions, unexpected student responses, difficulties, and my emerging interpretations. During analysis, I repeatedly returned to the original interview transcripts, observation notes, students' written work, and reflective records rather than relying on my initial impressions. I deliberately retained evidence that challenged emerging interpretations, including instances where students relied on visual appearance or measurement without sufficient mathematical explanation. This reflexive process helped me maintain critical distance from my classroom experiences while using my contextual knowledge to interpret students' developing conceptual understanding.

5.4 Instructional Intervention

The intervention was conducted over 15 instructional days during regular Grade 10 mathematics lessons on circle theorems, with one 45-minute lesson conducted each day. I prepared GeoGebra

applets around the main concepts addressed in the lessons, including central and inscribed angles, the angle in a semicircle, angles in the same segment, cyclic quadrilaterals, and related relationships involving chords and arcs. The activities were not implemented as a fixed sequence. Instead, I adjusted the instructional focus across three action-research cycles in response to what I observed in students' learning.

In the first cycle (Days 1–5), I focused on familiarizing students with GeoGebra and helping them notice geometric relationships through dynamic representations. Students explored basic controls, moved points when opportunities were available, measured angles, and compared different configurations. I noticed, however, that some students focused more on the numerical values displayed on the screen than on the relationships underlying those values. This became an important point for reflection. In response, I redesigned subsequent activities to include questions such as *"What remains unchanged when the point is moved?" "What do you notice about the two angles?"*, and *"How can you describe the relationship you are seeing?"*

In the second cycle (Days 6–10), the activities focused on angles in the same segment and cyclic quadrilaterals. Students compared several configurations, discussed their observations, and formulated tentative conjectures. Although students were becoming more comfortable with the dynamic environment, I observed that some still treated repeated measurements as sufficient evidence for a theorem. I therefore shifted the emphasis from *"What is the value?"* to questions such as *"Will this relationship remain true if the point is moved?" "Can you find another configuration?"*, and *"How can you convince your group that the relationship is always true?"* Students discussed their observations in four groups and were encouraged to explain, question, and challenge one another's ideas.

In the third cycle (Days 11–15), the focus shifted from exploration and conjecture toward mathematical explanation, justification, and application. Students were asked to use their earlier observations as a starting point for explaining why the relationships held. The activities therefore required them to provide reasons for their answers, connect dynamic observations with formal mathematical statements, and apply circle-theorem relationships to problems beyond the immediate GeoGebra configurations. The emphasis was increasingly placed on moving from *seeing that a relationship holds to explaining why it holds*.

Thus, the three cycles represented a responsive process rather than simply a progression in task difficulty. Each cycle addressed a specific learning issue identified through reflection on the preceding cycle: limited attention to underlying relationships in Cycle 1, reliance on measurement as evidence in Cycle 2, and the need to connect dynamic observation with mathematical justification in Cycle 3. Table 2 summarizes the focus of each cycle, the learning issue identified, and the corresponding instructional adjustments.

Table 2 Action-research cycles and instructional development during the 15-day intervention

Cycle	Days	Main focus	What students did	What I observed	Instructional adjustment
Cycle 1: Exploring and visualizing	1–5	Introduction to GeoGebra; central and inscribed angles; angle in a semicircle	Students explored the applets, moved points when opportunities were available, measured angles, and compared changing configurations.	Some students focused mainly on numerical measurements and found it difficult to describe the underlying geometric relationship.	I introduced more focused questions about what remained unchanged and asked students to describe and discuss the relationship rather than only report measurements.
Cycle 2: Exploring, discussing, and forming conjectures	6–10	Angles in the same segment; cyclic quadrilaterals	Students compared configurations, discussed observations in groups, tested possible relationships, and formulated initial conjectures.	Some students still treated repeated measurements as sufficient evidence that a theorem was true.	Tasks were revised to require students to test different configurations, explain whether the relationship remained valid, and give reasons for their conjectures.
Cycle 3: From observation to justification	11–15	Cyclic quadrilaterals and related applications; mathematical justification	Students connected GeoGebra observations with mathematical explanations, solved circle-theorem problems, and justified their answers.	Students were increasingly able to explain observed relationships, but some still needed support in connecting visual evidence with formal reasoning.	Greater emphasis was placed on *why* relationships held, requiring explanations, reasons for solution steps, and application to unfamiliar or examination-type problems.

5.4.1 Pre- and Post-Intervention Assessment

A 10-mark pre-test was administered on November 25, 2024, to assess students' prior understanding of circle-related concepts. Of the 32 students, 29 completed the pre-test, while all 32 completed the post-test administered on December 11, 2024, after the 15-day intervention.

Therefore, the paired comparison was based on the 29 students who completed both tests. As presented in Table 3, the mean score increased from 3.17 (SD = 1.69) in the pre-test to 6.12 (SD = 1.38) in the post-test, indicating a mean increase of 2.95 points. These results are interpreted descriptively alongside the qualitative evidence rather than as a controlled measure of GeoGebra's effectiveness. The assessment tasks, the marking scheme, and representative items are provided in Appendix A.

Table 3 Individual Paired Pre- and Post-Test Scores of Grades 10 Students

Student ID	Pre-test	Post-test	Change
S01	2.5	5.0	+2.5
S02	1.5	4.5	+3.0
S03	3.5	6.0	+2.5
S04	2.0	5.5	+3.5
S05	2.0	6.0	+4.0
S06	2.0	5.0	+3.0
S07	4.5	7.0	+2.5
S08	2.0	5.0	+3.0
S09	1.5	5.0	+3.5
S10	2.5	5.5	+3.0
S11	0.5	4.0	+3.5
S12	2.5	5.0	+2.5
S13	2.0	6.0	+4.0
S15	3.0	6.0	+3.0
S16	3.5	6.5	+3.0
S17	5.5	6.5	+1.0
S18	3.5	7.0	+3.5
S19	4.0	8.0	+4.0
S21	2.5	5.0	+2.5
S22	3.0	7.0	+4.0
S23	2.0	5.5	+3.5
S24	4.0	6.0	+2.0
S25	2.0	5.0	+3.0
S26	4.5	7.5	+3.0
S27	3.5	6.0	+2.5
S29	6.5	8.0	+1.5
S30	8.0	10.0	+2.0
S31	1.5	5.0	+3.5
S32	6.0	9.0	+3.0
Mean (SD)	3.17 (1.69)	6.12 (1.38)	2.95 (0.74)

The descriptive increase in mean scores indicates improved performance following the intervention. However, this improvement cannot be attributed to GeoGebra alone, as the study did not include a comparison group and the assessment was designed and scored by the teacher-researcher. The pre- and post-test results are therefore interpreted as supporting evidence of changes in students' performance and are considered alongside the qualitative findings from classroom observations, students' written work, interviews, and reflective responses.

5.4.2 Instructional Setting and Technology Access

The intervention was conducted in the regular Grade 10 mathematics classroom. The classroom had a permanently installed overhead projector, which was used to display the GeoGebra applets I prepared for the circle-theorem lessons. I used my personal laptop to run and demonstrate the applets, while my personal mobile phone was used to record selected audio and video during the intervention. No additional computers, tablets, or mobile devices were available for individual student use. Consequently, students did not have continuous or individual access to GeoGebra. Most activities were conducted through whole-class projection, in which I manipulated points, angles, arcs, and measurements while students observed the changes, made predictions, responded to questions, and discussed their interpretations. During selected practical activities, students worked in four groups and had opportunities to interact directly with the available GeoGebra activities. In these activities one student from each group operated the laptop at a time, while the other group members observed the dynamic representation, discussed their observations, and suggested actions; the opportunity to operate the laptop was then rotated among group members. Students directly manipulated the software on 6 of the 15 instructional days. On the remaining days, I operated the laptop and projected the GeoGebra applets for the whole class, while students observed, discussed, and responded to questions. This distinction is important because students' engagement with GeoGebra involved both teacher-operated demonstrations observed collectively by the class and limited opportunities for direct, shared student manipulation. Thus, the intervention did not provide

continuous individual access to GeoGebra; rather, students experienced the technology through a combination of teacher-guided demonstration, collective observation and discussion, and rotating group-based hands-on exploration.

The GeoGebra applets were created by me before the lesson delivery, and these were specifically created for the circle-theorem topics covered in the intervention. The applets were available for offline use, so internet access was not required during the lessons. Electricity was generally reliable throughout the intervention, and no significant electricity-related interruption occurred during the 15 days. This technology arrangement is important for understanding the nature of the intervention: the study did not involve a one-device-per-student model, but examined how dynamic visualization could be used within the actual technological conditions of the classroom. At the beginning of the intervention, the students had no prior experience with GeoGebra, and some reported that they had not even heard of the software before. The initial lessons therefore included a brief orientation to the GeoGebra environment and demonstrations of basic features such as moving points, observing angles and measurements, and examining changes in geometric configurations. As students became more familiar with the applets, I encouraged them to predict what might happen before a figure was manipulated, describe what they noticed afterward, and discuss their observations with their classmates. During the selected group activities, students were also given opportunities to manipulate the available applets directly. Because access to the laptop was shared, however, direct manipulation was necessarily limited and varied across activities. The projected GeoGebra environment consequently served as a shared visual space through which students could observe, question, compare, and reason about geometric relationships together.

5.4.3 Classroom Interaction and Collaboration

The teacher's role was not limited to operating the software. I first introduced and explained the relevant circle-theorem concepts and then used GeoGebra applets to provide dynamic representations of the relationships under discussion. During the activities, I asked questions about the theorem, observed changes, and the reasons underlying mathematical relationships. For example, students were asked what happened when a point was moved, which quantity remained unchanged, whether the same relationship could be observed in different configurations, and how the observation could be explained mathematically. Students responded through classroom discussion, observation, and practical activities, allowing their initial visual observations to become topics for mathematical discussion and explanation.

During selected practical activities, students were organized into four groups and worked together to examine geometric figures, compare configurations, and discuss the relationships represented in the applets. They shared interpretations, asked questions, and explained their ideas to one another, particularly when they were uncertain about how an applet worked or how an observed relationship was connected with a theorem. One student from each group operated the laptop at a time, while other members observed and discussed the emerging representation, with opportunities for manipulation rotated among group members. On these occasions, the GeoGebra representation provided a common visual reference around which students could compare observations and consider whether a relationship remained unchanged. I moved between groups, asked probing questions, clarified misunderstandings, and encouraged students to explain the reasons behind their observations.

Throughout the intervention, I took the role of a facilitator who guided students' exploration through questioning, explanation, and discussion rather than relying solely on direct transmission of procedures. These interactions were informed by Vygotsky's sociocultural perspective, particularly the ideas of mediation, social interaction, and guided participation. GeoGebra functioned as a mediating tool through which students could explore mathematical relationships, while teacher questioning and peer discussion supported them in articulating, comparing, and refining their mathematical ideas. However, because the GeoGebra activities were conducted mainly through a teacher-operated laptop and classroom projection, opportunities for direct student manipulation were limited and shared rather than individual. This distinction is important when interpreting students' engagement with the technology and the subsequent findings.

5.5 Data Collection

To understand students' learning experiences from different perspectives, data were collected from several sources throughout the 15-day intervention. These included classroom observations, semi-structured interviews, students' written reflections and worksheets, and the researcher's reflective journal. Using these different sources allowed us to compare what students said, what they did during classroom activities, what they produced in their written work, and what the researcher observed and reflected on. This helped us develop a more complete understanding of

students' developing mathematical reasoning and the challenges they experienced while learning circle theorems through GeoGebra.

5.5.1 Classroom Observations

Classroom observations were carried out throughout the intervention. During the lessons, the researcher paid attention to how students interacted with the GeoGebra activities, responded to questions, discussed mathematical ideas, and interpreted the changing geometric figures. Particular attention was given to students' ways of reasoning, their use of visual information, their attempts to identify invariant relationships, and the difficulties they experienced when moving from what they observed to explaining why a relationship held. Brief field notes were recorded during and after the lessons to capture important classroom episodes and students' responses.

5.5.2 Semi-Structured Interviews

Semi-structured interviews were conducted with the six focal students after the intervention to explore their experiences and perspectives in greater depth. The interviews focused on students' experiences with GeoGebra, their understanding of circle-theorem relationships, the role of dynamic visualization in their learning, and how teacher questioning and peer interaction supported or challenged their thinking. The interviews also provided opportunities to explore difficulties and experiences that were not fully visible through classroom observations. Interviews were conducted in Nepali, audio-recorded with participants' permission, and transcribed in Nepali. The relevant excerpts were translated into English by the first author for reporting in this article, with attention to preserving the intended meaning and context of students' responses. The interviews were conducted after completion of the 15-day intervention, between December 11, 2024 and January 15, 2025, and each interview lasted approximately 45 to 60 minutes.

5.5.3 Students' Reflective Responses and Worksheets

Students' written reflections, classroom activities, and worksheets were collected throughout the intervention. These materials were examined to understand how students expressed mathematical relationships in their own writing and how their reasoning developed during the activities. Particular attention was given to students' explanations, diagrams, use of mathematical terms, recognition of relationships, and instances of incomplete or incorrect reasoning. The written work was considered together with classroom observations and interview responses rather than being treated as a separate measure of achievement.

5.5.4 Researcher's Reflective Journal

The researcher maintained a reflective journal throughout the intervention. The journal was used to record classroom experiences, instructional decisions, students' difficulties, unexpected responses, and reflections after each lesson and action-research cycle. It also documented how observations from one cycle informed changes in the following cycle. In this way, the journal helped us trace the relationship between what happened in the classroom, how we interpreted those experiences, and how subsequent instruction was adjusted.

5.6 Data Analysis

The qualitative data were analyzed using thematic analysis following Braun and Clarke's (2021) approach. The analysis was mainly inductive, meaning that the themes were developed from what students said, did, and experienced during the GeoGebra-supported lessons rather than being fixed before the analysis. Data from classroom observations, semi-structured interviews, student worksheets and reflections, and the researcher's reflective journal were considered together to understand how students' mathematical thinking developed during the 15-day intervention. The analysis began with repeated reading of the interview transcripts, classroom observation notes, student work, and reflective journal entries. This helped the researcher become familiar with the data and identify important episodes related to students' mathematical learning. During the initial coding, meaningful parts of the data were given descriptive codes. Some of the initial codes included recognizing invariant relationships, dragging points to test a conjecture, explaining a geometric relationship, relying on visual appearance, asking for clarification, explaining ideas to peers, testing different configurations, connecting observations with a theorem, and difficulty moving from observation to mathematical justification.

The initial codes were then compared across the different data sources. Codes that expressed similar ideas were grouped into broader categories. For example, codes related to dragging points, testing different configurations, and noticing that an angle relationship remained unchanged were grouped under dynamic exploration and invariant recognition. Similarly, codes related to peer expla-

nation, discussion, and collaborative interpretation were grouped under peer-mediated mathematical reasoning. After this process, candidate themes were developed by examining the relationships among the categories and considering their connection with the research question. The candidate themes were reviewed again against the complete dataset to check whether they were clearly supported by the evidence and were sufficiently different from one another. During this process, the researcher also considered evidence that did not support the initial positive interpretation. For example, some students relied mainly on visual appearance or repeated measurements without being able to explain why a relationship remained true. Such cases were retained in the analysis because they helped show the difficulties students experienced in moving from visual observation to mathematical reasoning.

The final themes were developed by identifying the main idea represented by each theme and examining how it explained students' developing mathematical understanding during the GeoGebra-supported activities. Evidence from interviews was compared with classroom observations, student worksheets, student reflections, and the researcher's reflective journal. This comparison helped identify both similarities and differences among the sources. Therefore, triangulation was used not only to support the findings but also to examine whether different sources confirmed, qualified, or challenged the researcher's interpretations. The researcher, who also served as the mathematics teacher, conducted the initial coding and development of the themes. Because of this dual role, the researcher maintained a reflective journal throughout the intervention to record classroom observations, instructional decisions, students' difficulties, and emerging interpretations. The coded data were repeatedly compared with the original transcripts, student work, observation notes, and journal entries. This process helped the researcher reconsider early interpretations and avoid relying only on evidence that supported the expected positive outcomes. The coding process is illustrated in Table 4.

Table 4 Examples of coding and development of themes

Example from the data (anonymized extract)	Data source	Cycle	Initial code	Broader category	Final theme
"Previously I was unable to identify the relationship between the inscribed and central angles. Now I can easily identify it because of the teacher's clear explanation using the GeoGebra applet." – Bibek	Interview	Cycle 3	Recognizing the relationship between angles	Developing conceptual understanding	Developing Conceptual Understanding Through Dynamic Visualization
"No, sir. BC is a chord, but if it passes through the centre, then it becomes a diameter." -Yesh	Interview	Cycle 2	Distinguishing chord and diameter	Connecting visual features with mathematical definitions	Developing Conceptual Understanding Through Dynamic Visualization
"The central angle $\angle BAC$ (90.2°) is twice the inscribed angle $\angle BDC$ (45.1°)." -Manish	Classroom interaction	Cycle 1	Relating central and inscribed angles	Mathematical reasoning and theorem application	Moving from Dynamic Observation to Mathematical Reasoning and Justification
"I had no idea math could be so much fun-the way we can see the angles change and still stay equal is amazing!" -Grishma	During Class	Cycle 2	Relying on visual appearance	Visual evidence without justification	Moving from Dynamic Observation to Mathematical Reasoning and Justification
"As I moved all of them, and it's always doubling! Which is amazing." -Prayag	Observation	Cycle 1	Questioning an invariant relationship	Developing mathematical reasoning	Moving from Dynamic Observation to Mathematical Reasoning and Justification
"Sir, I think I get it now. If the opposite angles in a cyclic quadrilateral always add to 180°, that means even if I change the shape, the rule does not change!" - Nishu	Interview	Cycle 3	Explaining or challenging an idea	Peer mathematical discussion	Peer Interaction and Teacher Scaffolding in Mathematical Reasoning

The coding process is illustrated in Table 4 using anonymized extracts drawn directly from the dataset. Each extract is linked to its data source and the action-research cycle in which it was generated. The extracts were coded initially at the level of students' observed actions, statements, or explanations and were then grouped into broader categories before being developed into the three final themes.

5.7 Indicators of Conceptual Understanding

In this study, conceptual understanding was examined through students' mathematical reasoning across classroom activities, GeoGebra explorations, discussions, worksheets, interviews, and written reflections. The teacher focused particularly on how students made sense of geometric relationships as they manipulated and explored different configurations. Five indicators were used to identify evidence of conceptual development: (a) recognizing invariant relationships across changing configurations; (b) explaining mathematical relationships rather than only describing vi-

sual observations; (c) connecting dynamic representations with mathematical statements, diagrams, or symbolic reasoning; (d) applying circle-theorem relationships to unfamiliar or non-routine problems; and (e) moving from empirical observation and measurement toward mathematical explanation and justification. These indicators were applied as four distinct levels of evidence – recognition of a relationship, correct application of a relationship, explanation of a relationship, and justification of why it holds – and these levels are not treated as interchangeable in the reporting of the findings.

The post-intervention assessment was also considered as complementary evidence, particularly when students' written responses demonstrated their ability to apply circle-theorem relationships, explain geometric relationships, or transfer their understanding to different configurations. However, assessment scores were interpreted together with the richer qualitative evidence rather than treated as a direct measure of conceptual understanding. These indicators also helped us distinguish conceptual understanding from general classroom engagement. Students' enjoyment, confidence, willingness to participate, and positive attitudes toward GeoGebra were treated as evidence of the learning experience rather than as direct evidence of conceptual understanding. For example, when a student noticed that an angle remained unchanged while a point was dragged, the teacher initially treated this as evidence of observation or conjecture. Stronger evidence of conceptual understanding was considered when the student could explain why the relationship remained unchanged and apply the idea in another configuration. We also paid attention to episodes in which students initially relied on visual appearance or repeated measurements but later revised their thinking through teacher questioning, peer discussion, or further exploration. In this way, the analysis considered both successful learning episodes and the difficulties students experienced in moving from visual observation toward mathematical explanation.

5.8 Trustworthiness

Several strategies were used to strengthen the trustworthiness of the study. First, methodological triangulation was employed by comparing evidence from classroom observations, students' written work and reflective responses, semi-structured interviews, the diagnostic pre-test and post-test, and the researcher's reflective journal. Second, prolonged engagement over the 15-day intervention enabled the teacher-researcher to observe students' learning experiences across multiple instructional cycles rather than relying on a single classroom episode. Third, reflexivity was maintained through a reflective journal in which the first author documented instructional decisions, unexpected student responses, emerging interpretations, and potential influences of the dual role of teacher and researcher. During analysis, the first author repeatedly returned to the original transcripts, observation notes, and students' written work and considered evidence that challenged emerging interpretations, including instances in which students relied primarily on visual appearance or repeated measurement without sufficient mathematical explanation. Member checking was not conducted. After the initial analysis, the interpretations were not returned to the six focal students for confirmation or revision. This limitation is acknowledged because the first author was both the classroom teacher and the primary interpreter of the data. Nevertheless, the use of multiple data sources, prolonged engagement, and reflexive analysis provided opportunities to compare interpretations across different forms of evidence and to maintain close connection between the findings and the students documented learning experiences.

5.9 Ethical Considerations

Before beginning the intervention, written permission was obtained from the school administration to conduct the study in the Grade 10 classroom. The students were informed about the purpose of the study, the activities involved, and the voluntary nature of their participation, and their oral assent was sought before participation. Oral consent was also obtained from the students' parents or guardians before the intervention. Parental consent was obtained orally rather than through individual written consent forms; this description reflects the procedure actually followed and is not attributed to class size or any other practical consideration. Students were informed that participation in the semi-structured interviews and the use of their classroom work and responses for research purposes were voluntary and that they could decline to participate without affecting their regular learning. Pseudonyms were used in reporting the findings, and identifying information was removed from the reported data to protect participants' identities. The research activities were conducted within the normal classroom setting and were integrated into the students' regular mathematics lessons rather than requiring participation in activities unrelated to their routine learning. The first author was responsible for explaining the study to the students and obtaining oral student assent and oral consent from their parents or guardians before data collection. The consent procedures covered participation in interviews, audio and video recording, the use of students'

direct quotations, and the reproduction of students' handwritten work. Participation was voluntary, and students were informed that they could decline to participate or withdraw from the study without any academic consequences. The study received ethical approval from the Kathmandu University School of Education (KUSOED) before the intervention and data collection began.

6 Findings

The findings draw on classroom observations, students' reflective responses, interview transcripts, worksheets, and the researcher's reflective journal collected throughout the 15-day GeoGebra-supported intervention. Through thematic analysis, we examined how students noticed, interpreted, discussed, and explained geometric relationships as they engaged with circle theorems. The analysis revealed a gradual movement from dynamic observation toward conceptual understanding and mathematical justification, shaped by students' interactions with the GeoGebra representations, their peers, and the teacher's questioning. The findings are organized around three interconnected themes: developing conceptual understanding through dynamic visualization; moving from dynamic observation to mathematical reasoning and justification; and the role of peer interaction and teacher scaffolding in mathematical reasoning. Together, these themes illuminate not only the opportunities GeoGebra created for students to explore and make sense of geometric relationships, but also the pedagogical support required to transform visual exploration into meaningful mathematical understanding.

6.1 Developing Conceptual Understanding Through Dynamic Visualization

Students' understanding of circle theorems began to change when they were able to see and explore the relationships rather than only read them from the textbook. In the early lessons, many students were familiar with terms such as radius, chord, arc, sector, and segment, but their understanding was often based on remembering definitions and identifying them in fixed diagrams. When the GeoGebra applets were introduced, students were given opportunities to move points, change the figures, observe what happened, and compare different configurations. This created a different way of learning in which the geometric relationships were not presented as fixed rules but could be explored and discussed.

For instance, while exploring the basic elements of a circle, the students observed how the teacher manipulated the figures and the changes that occurred from that; during selected group work, they had also opportunities to manipulate the applets. These activities helped students notice relationships that were difficult to see clearly in static textbook diagrams. More importantly, students gradually began to pay attention to what remained unchanged when the figures were changed. This recognition of invariant relationships became an important part of their developing conceptual understanding. The classroom observations also showed that students gradually became more willing to question what they saw and ask why a particular relationship remained unchanged. Nishu, for example, actively discussed the differences between chords, sectors, and segments during the activities. Similarly, Grishma and Prayag frequently asked questions about the relationships shown in the GeoGebra applets and tried different configurations to see whether the same relationship continued to hold. These episodes suggested that students were moving beyond simply identifying geometric objects toward examining the relationships among them. The change in students' experiences was also visible in their written reflections. Beyond what we observed during classroom activities and heard in interviews, students themselves described how the visual and animated features of GeoGebra helped them approach mathematical ideas differently. One student, for example, Prayag, described GeoGebra as a *"good technique for learning"* and specifically highlighted its animations and colorful representations. The student further reflected that GeoGebra helped in developing *"conceptual understanding of theorems of circles"* and expressed a willingness to use it for learning other mathematical topics. This reflection is particularly meaningful when read alongside our classroom observations. The student was not simply commenting on the appearance of the software; the reflection connected the visual features of GeoGebra with understanding mathematical concepts. At the same time, we interpret this evidence cautiously. A student's positive perception does not, by itself, demonstrate conceptual mastery. Rather, it provides an important insight into how students experienced the learning environment and how dynamic visualization was perceived as supporting their mathematical sense-making.

The handwritten reflection shown in Figure 1 further illustrates this student perspective. It captures, in the student's own words, how the visual and animated representations of GeoGebra were experienced as different from conventional mathematics learning. Such reflections complemented the interview accounts, classroom observations, and students' written mathematical work,

helping us understand not only what changed in the classroom, but also how students themselves experienced that change.

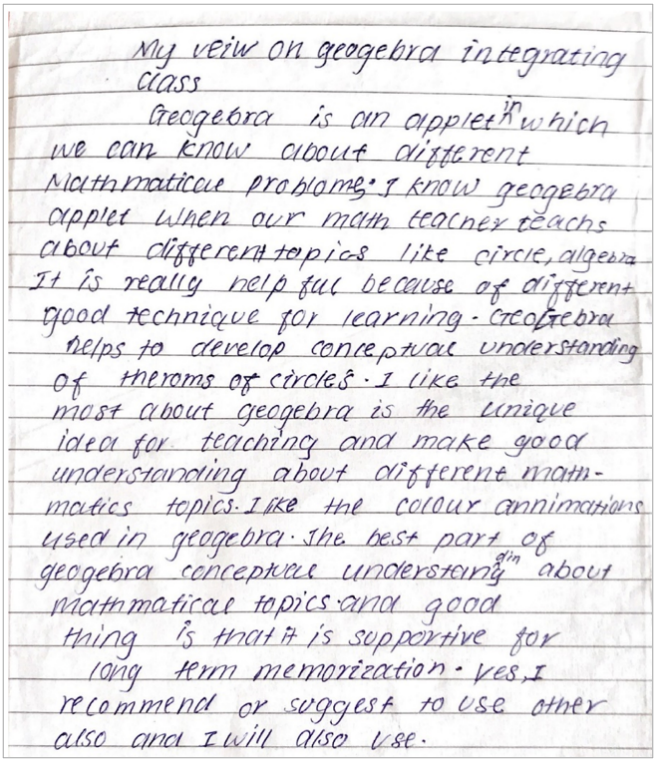

My veiw on geogebra integrating class
Geogebra is an applet in which we can know about different Mathmaticae problems. I know geogebra applet when our math teacher teachs about different topics like circle, algebra It is really help ful because of different good technique for learning. Geogebra helps to develop conceptual understanding of theroms of circles. I like the most about geogebra is the unique idea for teaching and make good understanding about different mathmatics topics. I like the colour annimations used in geogebra. The best part of geogebra conceptual understanding about mathmaticae topics and good thing is that it is supportive for long term memorization. Yes, I recommend or suggest to use other also and I will also use.

Figure 1 Student reflection on GeoGebra-supported learning (The handwritten reflection illustrates a student's perception of how GeoGebra's visual and animated features supported learning and understanding of circle theorems.)

Students' own reflections also showed how this way of learning differed from their earlier experiences. Manish explained that diagrams helped him understand some mathematical ideas that were difficult to understand from formulas alone. Yesh similarly described the opportunity to interact with mathematical objects as more enjoyable than simply memorizing rules. He remarked, *"Today I understood easily, sir. First time I am enjoying learning mathematics, sir."* While this comment mainly reflects Yesh's positive learning experience, it also provides context for understanding why he was willing to engage more actively with the mathematical ideas presented through the applets. A more important indication of conceptual development was seen when students began connecting their observations with mathematical relationships. Rather than accepting a relationship simply because it appeared on the screen, some students started asking whether it remained true when the figure was changed and what mathematical reason could explain it. In this sense, GeoGebra did not by itself provide conceptual understanding. Instead, the dynamic figures created opportunities for students to explore relationships, notice invariants, raise questions, and develop explanations. The teacher's questioning and classroom discussion were important in helping students move from what they could see toward what they could explain mathematically. Thus, the findings suggest that dynamic visualization provided a useful bridge between students' visual experiences and their developing conceptual understanding of circle theorems. The value of GeoGebra was not simply that it made mathematics more interesting, but that it allowed students to interact with changing geometric representations and examine relationships across different configurations. At the same time, the classroom evidence showed that visual exploration alone was not always sufficient. Some students initially relied on what they could see or measure, which created a need for further questioning and explanation. This movement from visual observation toward mathematical reasoning became an important feature of students' conceptual development during the intervention.

An important change observed during the intervention was that some students who were usually less active in mathematics lessons began to participate more openly. One student who rarely spoke during mathematics classes reflected, *"Earlier, we were just memorizing theorems. Now we get to see them working."* This response is important because it shows a change in how the student experienced the learning process. Rather than receiving theorem statements as fixed rules, the student was able to see and interact with the relationships represented in the GeoGebra applets.

Although increased participation alone cannot be taken as evidence of conceptual understanding, it created more opportunities for students to ask questions, test ideas, and discuss mathematical relationships. The dynamic nature of GeoGebra also helped sustain students' attention while they explored relationships among different circle theorems. For example, students were surprised to observe that an angle relationship remained unchanged even when the position of a point on the circumference was changed. Such moments encouraged students to ask questions such as why the relationship remained unchanged and whether it would continue to hold in other configurations. These responses were more meaningful for our analysis than students' expressions of enjoyment because they showed students beginning to notice mathematical invariance and explore the reason behind it. In this way, curiosity became connected with mathematical exploration rather than remaining simply a response to the use of new technology.

This finding can be understood through Vygotsky's sociocultural perspective, particularly the idea that students develop understanding through interaction with tools, other people, and the learning environment (Vygotsky, 1978). In the present study, GeoGebra served as a mediating tool that made changing geometric relationships visible, while teacher questioning and peer discussion helped students interpret what they observed. The role of the teacher was therefore not limited to demonstrating the applets. Questions about what changed, what remained unchanged, and why the relationship held encouraged students to move from observation toward mathematical explanation. This interpretation is also consistent with the earlier conceptualization of GeoGebra in this study as a mediating resource that supported students within their developing mathematical understanding. The finding is broadly consistent with previous studies of GeoGebra-supported mathematics learning. Thapa et al. (2022), working with circle concepts in the Nepalese school context, reported that GeoGebra encouraged students to explore geometric relationships through visual and interactive representations. Similarly, research on dynamic geometry has shown that opportunities to manipulate mathematical objects and receive immediate visual feedback can support students' active involvement in learning (Hohenwarter & Jones, 2007; Arbain & Shukor, 2015; Zulnaidi & Syed Zamri, 2017). Our findings add a more specific perspective by showing that students' increased participation was closely connected with opportunities to observe, question, and test geometric relationships during the action-research intervention.

6.2 From Dynamic Observation to Mathematical Justification

A second important pattern in the data was the gradual movement of students from simply observing geometric relationships to making conjectures, explaining those relationships, and attempting to justify their mathematical reasoning. At the beginning of the intervention, many students were more familiar with learning geometry by remembering theorem statements and applying them to familiar diagrams. Their classroom responses suggested that they could often recall a rule but found it difficult to explain why the relationship held. The use of GeoGebra created opportunities to look at these relationships differently. During the GeoGebra-supported activities, students explored geometric relationships through a combination of teacher-guided demonstrations, whole-class discussion, and limited shared hands-on interaction. I manipulated points, arcs, and angles on the projected applets and invited students to predict what might happen before the changes were made. During selected group activities, students also had opportunities to interact directly with the applets. Although this access was limited by the availability of a single laptop, the dynamic changes visible to the whole class became a common object for observation, questioning, and discussion.

During selected group activities, students manipulated points on the circumference and observed that the inscribed angle remained unchanged. Other students followed the manipulation through the projected display and participated in discussing the observed relationship. For instance, when I dragged points around the circumference on the projected GeoGebra applet, students observed that the inscribed angle remained unchanged and discussed why the relationship appeared to hold across different configurations. During selected group activities, students also had opportunities to manipulate the points themselves. Rather, they provided students with empirical evidence from which they could formulate and test conjectures. The important learning process occurred when students were encouraged to move beyond statements such as *"the angle remains the same"* and to ask why the relationship remained invariant across different configurations.

The change was also visible in students' written mathematical work. In the earlier activities, some students could identify a relationship from the diagram or measurement but found it difficult to explain why the relationship remained true. In later activities, their written responses increasingly connected the observed relationship with the relevant circle theorem and included mathematical reasoning. This movement from noticing a relationship to explaining and applying it provided an important strand of evidence for developing conceptual understanding. The experiences of the

focal students illustrate this movement. During an activity on inscribed angles, Bibek observed that when the central angle was 180 degrees, the corresponding inscribed angle was 90 degrees. Bibek reflected on his earlier difficulty with the concept:

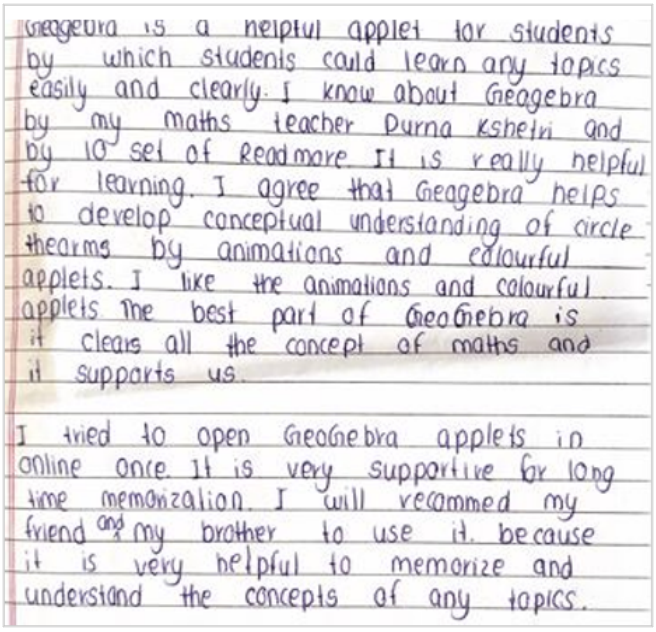
"Previously, I could not identify the relationship between the central angle and the inscribed angle easily. Now

I can identify it easily because of sir's clear explanation using the GeoGebra applet."

His reflection suggests that the dynamic representation, together with the teacher's explanation, helped him see the relationship more clearly and connect the visual representation with the underlying mathematical concept. His observation represented an initial conjecture based on dynamic exploration rather than simply recalling a previously memorized theorem. Similarly, while exploring inscribed angles, students noticed that the relationship remained unchanged as points were moved. One student responded, *"No change, sir."* Such responses provided early evidence of students noticing invariants through dynamic exploration. However, as our analysis showed, noticing that a relationship remained unchanged was treated as an initial step toward conceptual understanding rather than proof of it. Similarly, Grishma questioned whether the same relationship would remain valid when the position of the inscribed angle was changed around the circle. Her question was important because it moved the discussion from a single observed case toward consideration of whether the relationship was invariant across configurations. At the same time, the data also showed that visual exploration did not automatically lead to mathematical justification. Some students initially accepted a relationship because it appeared to remain unchanged on the screen or because repeated measurements produced the same result. This became an important point of reflection during the intervention. In response, the researcher increasingly used questions such as *"What remains unchanged?"*, *"Why do you think this relationship holds?"*, and *"Would this relationship always be true?"* Students were encouraged to explain their observations rather than treating repeated measurements as sufficient evidence. In this sense, teacher questioning and classroom discussion became important in helping students move from empirical verification toward mathematical explanation.

This transition can also be seen in students' reflections. Yesh explained that the dynamic representation helped him understand the reasoning behind a theorem rather than simply remember its statement. Nishu similarly described the geometric relationships as becoming clearer through the visual representations. Manish noted that the applets helped him concentrate on the relationships involved in explaining circle theorems. These accounts suggest that the value of the dynamic environment was not simply that students could see a theorem, but that they could interact with a representation and use it as a basis for questioning and explanation. This shift from viewing GeoGebra as a demonstration tool to using it as a means of making mathematical ideas visible was also evident in students' written reflections. Nishu, for example, wrote that GeoGebra helped her develop a "conceptual understanding of circle theorems" and that the animations and visual features made the concepts clearer. Her reflection suggests that dynamic visualization was beginning to support understanding beyond simply remembering theorem statements. This reflection was consistent with classroom observations in which Nishu increasingly engaged with the visual representations and discussions surrounding circle theorems. Importantly, however, her reflection represents her perception of learning rather than, by itself, evidence of conceptual mastery. (see Figure 2)

This reflection illustrates how the student perceived dynamic animations and visual representations as helping to make circle-theorem concepts clearer and more understandable. Evidence of developing understanding was also found when students applied previously explored relationships to problems in less familiar configurations. For example, after exploring the relationship between central and inscribed angles, students were able to determine an inscribed angle when the corresponding central angle was given as 90.2 degrees, obtaining 45.1 degrees. More importantly, some students were able to explain their answers by referring to the relationship they had explored rather than merely recalling the numerical answer. Similar reasoning was observed in activities involving cyclic quadrilaterals, equal arcs, angles subtended by diameters, and related circle-theorem problems. These episodes provided stronger evidence of conceptual development because students were using a relationship beyond the exact configuration through which they had first encountered it. During one classroom discussion, I asked Manish, one of the focal students, what the central angle $\angle$BAC would be if the inscribed angle $\angle$BDC measured 45.1°. He immediately responded, *"The central angle $\angle BAC$ (90.2°) is twice the inscribed angle $\angle BDC$ (45.1°)."* His response illustrates a movement beyond simply observing the dynamic figure toward applying the observed relationship mathematically. Rather than relying on the displayed measurement alone, Manish articulated the relationship between the two angles and used it to determine the unknown angle. Following the


Geogebra is a helpful applet for students by which students could learn any topics easily and clearly. I know about Geogebra by my maths teacher Purna Kshetri and by 10 set of Readmore. It is really helpful for learning. I agree that Geogebra helps to develop conceptual understanding of circle thearms by animations and colourful applets. I like the animations and colourful applets the best part of GeoGebra is it clears all the concept of maths and it supports us.

I tried to open GeoGebra applets in online once. It is very supportive for long time memorization. I will recommed my friend and my brother to use it. because it is very helpful to memorize and understand the concepts of any topics.

Figure 2 Student Reflection as Evidence of Developing Conceptual Understanding

levels of evidence set out in Section 2.7, this episode is an example of correct application rather than of mathematical justification. Manish used the relationship to determine an unknown angle, but he did not explain why the central angle is twice the inscribed angle, and the doubling of a measured value is therefore not described here as proof. (see Figure 3)

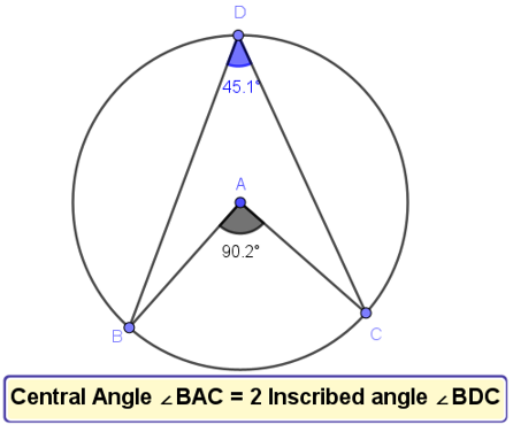


Figure 3 Dynamic GeoGebra visualization of the relationship between a central angle and an inscribed angle

The students' movement from observation to explanation was gradual. At first, some students relied mainly on what they could see or measure. Later, some began asking whether the same relationship would hold in other configurations and attempted to explain why. During the central-angle and inscribed-angle activity, Yesh reasoned, *"50 degrees, sir. Doubled it. Other circle theorems can also be dealt with in this way!"* This response showed that he was beginning to use an observed relationship beyond the original example. Importantly, not all students reached this level of explanation immediately. Some could identify an invariant through repeated measurement but could not yet explain its mathematical basis. We therefore interpreted such episodes as evidence of an emerging understanding rather than complete conceptual mastery. This distinction helped us avoid treating successful manipulation of GeoGebra as evidence of learning by itself. To illustrate how students' conceptual understanding developed during the intervention, the following examples show three focal students' initial responses or misconceptions, the GeoGebra-supported activities and teacher guidance they received, and their subsequent explanations and applications, including difficulties that remained. Initially, Nishu relied mainly on memorized formulas and static diagrams and indicated that clearer labels would help her understand the relationships in circle diagrams. During the GeoGebra activities, she was able to manipulate the figures and observe how changing one element affected the associated angles. With teacher questioning and attention to the changing

labels and measurements, she began to use the visual relationships rather than relying only on memorized rules. In later activities, she was able to explain circle-theorem relationships using the diagram and apply them to related problems. However, she still required occasional prompting when the relationship was presented in an unfamiliar configuration. Students' discussions with one another and the teacher's questioning became important when visual observation alone was insufficient. Rather than immediately providing explanations, I asked questions such as *"What do you notice?" "What remains unchanged?"*, and *"Why does it remain unchanged?"* These questions encouraged students to revisit their observations and explain their thinking. Although dynamic visualization supported conjecture formation and helped students recognize invariant relationships, some students continued to rely heavily on what they could see or measure. This suggests that dynamic geometry should not be treated as a substitute for deductive reasoning. Its contribution was more evident when visual exploration was followed by purposeful questioning, discussion, written explanation, and mathematical justification. Thus, the learning process observed in this study was not simply "seeing a theorem" through GeoGebra. It involved moving from seeing, to testing, to explaining, and, where possible, to justifying the observed relationship mathematically.

The findings suggest that dynamic visualization supported students' developing conceptual understanding not only through direct manipulation but also through teacher-guided demonstration, collective observation, mathematical discussion, and shared exploration. The assessment results provided an initial indication of change across the intervention. Among the 29 students who completed both assessments, the mean score increased from 3.17/10 before the intervention to 6.12/10 after it. However, the numerical change alone does not explain what students learned or how their thinking changed. The classroom evidence provided a more detailed picture. During the GeoGebra activities, students increasingly moved from describing what they could see to explaining why particular geometric relationships remained true. This interpretation is consistent with Vygotsky's (1978) sociocultural perspective, particularly the idea that learning is mediated through cultural tools and social interaction. In the present study, GeoGebra functioned as a mediating tool through which students could manipulate geometric objects and make relationships visible, while teacher questioning and peer discussion supported the development of mathematical explanations. Skemp's (1976) distinction between instrumental and relational understanding also helped us interpret the changes visible in students' responses. Early in the intervention, some students tended to rely on procedures, measurements, or memorized theorem statements. For example, when an angle remained unchanged after dragging a point, a response such as *"the measurement is the same"* indicated recognition of a result without necessarily explaining its mathematical basis. In contrast, when students began asking why the relationship remained unchanged, explaining the relationship to peers, or applying it to a new configuration, their responses showed movement toward relational understanding. We therefore use Skemp's distinction as an interpretive lens for understanding this movement from knowing *what to do* toward understanding *why it works*, rather than as a separate coding framework.

The findings are also broadly consistent with previous research indicating that dynamic geometry environments can support students' visualization, exploration, and understanding of mathematical relationships (Hohenwarter & Jones, 2007; Arbain & Shukor, 2015; Zulnaidi & Syed Zamri, 2017). However, the present classroom evidence adds an important qualification to this literature. GeoGebra did not independently produce mathematical justification. Instead, its educational value appeared to depend on how the dynamic representations were incorporated into classroom tasks and how the teacher responded to students' observations. In particular, the movement from visual evidence to mathematical reasoning required deliberate scaffolding. In sum, this theme suggests that the main contribution of GeoGebra in the present study was not simply to make circle theorems more visually attractive. Its greater value was in creating a space where students could manipulate configurations, notice invariants, test conjectures, question their observations, and gradually connect visual evidence with mathematical explanations. The findings therefore point to a progression from dynamic observation to conjecture, from conjecture to explanation, and from explanation toward mathematical justification. At the same time, the students' continued reliance on visual evidence in some situations shows that this progression cannot be assumed to occur automatically. It requires carefully designed tasks, purposeful teacher questioning, and opportunities for students to articulate and justify what they observe.

6.3 Peer Interaction and Teacher Scaffolding in Mathematical Reasoning

The third theme that emerged from the analysis was the role of peer interaction and teacher scaffolding in helping students make sense of the geometric relationships they encountered through GeoGebra. While the dynamic applets provided students with a visual environment for exploring

circle theorems, the classroom discussions that followed were equally important. Students did not always arrive at a mathematical explanation simply by manipulating the figures. In several episodes, they needed to explain what they had observed to their peers, listen to alternative interpretations, respond to questions, and reconsider their initial ideas. Thus, GeoGebra provided the shared visual representation, while interaction with peers and the teacher helped students turn their observations into mathematical reasoning.

This was particularly visible during activities in which students worked in small groups to explore different configurations. After manipulating points, arcs, and angles, students compared what they had observed and discussed whether the relationships remained valid when the configuration was changed. Such discussions gave students opportunities to express ideas that they might not have articulated during individual work. In some cases, one student's observation became the starting point for another student's question or explanation. GeoGebra functioned less as an individual learning device and more as a shared mediational resource. The projected dynamic representation gave students a common mathematical object to observe, question, and discuss, while teacher questioning and peer interaction helped them interpret what they saw. This suggests that the educational value of GeoGebra in a resource-limited classroom may not depend entirely on individual access to the software. Rather, dynamic visualization can become meaningful when it is embedded within purposeful dialogue, questioning, and instructional scaffolding. During a discussion about chords and diameters, some students initially focused on the appearance of the figure and assumed that a relationship would remain unchanged simply because it looked similar on the screen. Through questioning, the discussion moved beyond visual appearance. When the teacher asked whether the relationship would necessarily remain valid in every configuration, Yesh responded, *"No, sir. BC is a chord, but if it passes through the centre, then it becomes a diameter."* He further connected this observation with the relationship between a diameter and the angle subtended by it. This episode was significant because Yesh was not simply reporting what he could see. He was distinguishing between a general chord and a diameter based on a defining geometric property and connecting that distinction with a known theorem.

Teacher questioning played an important role in such moments. Rather than immediately providing the correct explanation, the teacher often used questions to encourage students to examine their assumptions and explain the reasons for their answers. Questions such as *"What do you notice?", "What has remained unchanged?", "Will this always happen?", and "How can you explain your answer?"* encouraged students to move from describing an observed result toward considering the mathematical relationship underlying it. This scaffolding was particularly important when students relied heavily on measurement or visual appearance. In such situations, the teacher's role was not simply to confirm whether an answer was correct, but to encourage students to provide a mathematical reason. The following GeoGebra activity shown in Figure 4 was created and shared as a peer interaction and teacher scaffolding in mathematical reasoning. As students moved the vertices of the cyclic quadrilateral, they could see that the opposite angles continued to add to 180○. They compared their observations, explained their ideas to peers, and questioned whether the relationship would remain true in other configurations. The teacher supported this process through questions such as, *"What do you notice when the point is moved?" and "Why do these two angles add to 180° ?"* Thus, GeoGebra provided the visual evidence, while peer discussion and teacher questioning helped students move from seeing a relationship to explaining why it holds. This suggests that the value of GeoGebra in this setting lay not in visualization alone, but in how visualization was combined with dialogue and purposeful scaffolding to support mathematical reasoning.

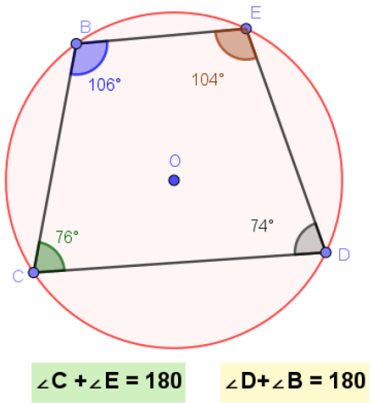


Figure 4 Dynamic GeoGebra representation of opposite angles in a cyclic quadrilateral

Peer explanation also became an important part of this process. Students sometimes explained an observed relationship to their classmates using their own words before the teacher introduced or formalized the mathematical statement. These interactions allowed students to hear different ways of explaining the same relationship and provided opportunities to question or clarify one another's reasoning. In this sense, collaboration was not treated as a separate intervention from GeoGebra. Rather, peer interaction was integrated into the GeoGebra-supported activities and provided a social setting in which students could interpret the visual evidence and communicate their mathematical ideas. The data also suggested changes in how some students approached unfamiliar problems. At the beginning of the intervention, students were often more hesitant when they encountered a geometric configuration that did not look exactly like the examples, they had previously studied. During later activities, some students appeared more willing to explore the configuration, discuss possible relationships, and test an idea before asking the teacher for confirmation. Manish, for example, explained that the GeoGebra representation helped him visualize the relationship before attempting to solve a theorem-based problem. Prayag similarly noted the usefulness of observing different configurations for identifying patterns. These accounts suggest that dynamic representations, combined with discussion and teacher support, gave students a starting point from which to approach unfamiliar problems. The classroom evidence also showed that students learned through sharing their interpretations. As one student, Nishu, remarked during the activities, *"Before, we used to memorize theorems. Now, we get to watch them happen!"* This statement captures an important change in the classroom experience, but it does not by itself establish conceptual learning. Rather, it shows how the shared visual environment created opportunities for students to talk about mathematics, while questioning and discussion helped move some of those observations toward explanation.

The role of confidence should, however, be interpreted carefully. Students' willingness to ask questions, share explanations, attempt unfamiliar problems, and discuss possible answers provided evidence of increased participation and mathematical confidence during the intervention. For example, Nishu reflected, *"I used to find geometry very difficult, but now I'm able to actually work on those problems."* Rather than treating this statement alone as evidence of increased conceptual understanding, it is better understood as evidence of a change in the student's perception of her ability to engage with geometry. This distinction is important because confidence and conceptual understanding are related but different aspects of learning. The findings also showed that productive interaction did not always occur automatically. Some students initially depended on the teacher to confirm their observations, while others were more comfortable accepting a visual result without questioning it. At times, peer discussions also remained at the level of reporting measurements rather than explaining mathematical relationships. These episodes became important points for teacher reflection. In response, subsequent activities placed greater emphasis on asking students to explain their reasoning, compare different configurations, challenge one another's interpretations, and provide reasons for their conclusions. In this way, teacher scaffolding gradually shifted from helping students operate the applets toward helping them reason with the mathematics represented through the applets.

Vygotsky's (1978) sociocultural perspective helped us interpret conceptual development as a socially mediated process rather than as an outcome of technology alone. In the classroom, GeoGebra acted as a mediating tool through which students could see and manipulate mathematical relationships, while teacher questioning and peer discussion supported them in making sense of what they observed. This was particularly visible when students could identify an invariant but were not yet able to explain why it remained unchanged. Through questions, discussion, and further exploration, some students gradually moved toward more complete mathematical explanations. We interpret these episodes through the idea of the Zone of Proximal Development (ZPD), where students were able to move beyond their initial explanations with appropriate guidance and interaction. Thus, the important learning process was not simply student–software interaction, but the interaction among student, teacher, peers, mathematical representation, and GeoGebra. GeoGebra gave students an opportunity to explore a configuration before committing themselves to a solution. They could test an idea, observe what happened, reconsider an assumption, and try another approach. This resembles the exploratory aspects of mathematical problem-solving described by Pólya and Conway (1945), particularly understanding the problem, developing a plan, examining possible relationships, and checking the resulting solution. However, the present findings suggest that the software itself did not create this problem-solving process. The classroom evidence suggests that repeated opportunities to explore, make mistakes, receive feedback, and discuss possible solutions may have contributed to students' willingness to engage with geometry. Overall, this theme highlights that GeoGebra-supported learning was a socially mediated process. The dynamic applets provided students with something concrete to observe and manipulate, but peer discussion and teacher scaffolding helped them interpret those observations and connect them

with mathematical ideas. The most meaningful learning episodes occurred when students were encouraged not only to say what they could see, but also to explain why they thought a relationship held. Thus, peer interaction and teacher questioning served as an important bridge between visual exploration and mathematical reasoning, while GeoGebra provided the dynamic representations around which that reasoning could develop.

7 Discussion

7.1 Interpretation of the Main Findings

This 15-day action-research study explored how GeoGebra-supported activities shaped Grade 10 students' learning of circle theorems. The findings suggest that dynamic visualization created meaningful opportunities for students to see and explore geometric relationships that were often difficult to understand through static diagrams alone. Students gradually moved from observing changing configurations to recognizing invariants, forming conjectures, explaining relationships, and applying their understanding to mathematical problems. The study also showed that learning was supported through interaction. Peer discussion and teacher questioning encouraged students to share ideas, compare observations, and develop their mathematical explanations. At the same time, the findings revealed an important limitation: visual evidence and repeated measurements did not automatically lead to conceptual understanding or mathematical proof. Some students initially relied on what they could see, and they needed purposeful questioning and scaffolding to move from empirical observation toward mathematical justification. Overall, the findings indicate that GeoGebra can serve as a useful mediating resource for making circle geometry more interactive, exploratory, and meaningful in the classroom. Its value, however, lies not in the software alone but in how it is integrated with carefully designed tasks, teacher scaffolding, peer interaction, and opportunities for students to explain and justify their mathematical thinking. Given the contextual nature of this study, the findings should be understood as insights into students' learning experiences within this particular Grade 10 classroom rather than as evidence of a causal effect of GeoGebra. Future research could examine these processes across different schools, learners, and longer periods of instruction.

7.2 Implications for Practice, Teacher Education, and Policy

The findings suggest several practical implications for mathematics teaching. First, GeoGebra can be used to create opportunities for students to explore geometric relationships rather than only memorize circle theorems. Teachers can design activities in which students manipulate figures, identify what remains unchanged, discuss their observations, and then connect these observations with formal mathematical reasoning. Second, the study highlights the importance of teacher scaffolding. Dynamic visualization alone does not guarantee conceptual understanding. Teachers need to ask purposeful questions that help students move from *"What do you see?"* to *"Why does it happen?"* and finally to *"How can you justify it?"* This connection between visual exploration and mathematical justification is particularly important when teaching geometry. Third, the findings have implications for mathematics teacher preparation and professional development. Teachers need opportunities to learn not only how to operate GeoGebra but also how to design meaningful tasks, anticipate students' misconceptions, and use questioning and peer discussion to support mathematical reasoning. Finally, the experience of using a teacher-operated laptop, projector, and prepared offline applets shows that technology-supported geometry lessons do not necessarily require one device for every student or continuous internet access. For schools in contexts such as Nepal, this provides a practical possibility for using freely available dynamic mathematics software within existing classroom resources. However, equitable student access, teacher preparation, and appropriate classroom support remain important considerations for effective implementation.

7.3 Limitations and Directions for Future Research

This study has several limitations that should be considered when interpreting the findings. First, it was conducted in one Grade 10 classroom in a single school in Nepal, with six students selected for detailed qualitative analysis. The 15-day intervention provided a close view of students' learning experiences, but the findings are not intended to be generalized to other contexts. Second, access to digital devices was limited. The intervention relied on one teacher laptop and a classroom projector, so students did not have continuous individual access to GeoGebra. Direct manipulation occurred mainly during selected shared activities, while much of the learning involved teacher-guided demonstrations, whole-class observation, questioning, and peer discussion. Thus, the findings should be understood as evidence of learning within a GeoGebra-supported classroom environment,

rather than as evidence of the effect of individual student use of the software. Third, because the first author was also the classroom teacher and researcher, his instructional decisions and expectations may have influenced classroom interactions and interpretation of the data. Reflexive documentation and comparison across different data sources were therefore used to make these influences visible during analysis. Finally, GeoGebra was used alongside teacher scaffolding, peer interaction, and inquiry-oriented activities; consequently, the study cannot isolate the independent contribution of GeoGebra to students' conceptual development. These limitations suggest several directions for future research. Because the pre- and post-tests were designed and scored by the teacher-researcher, the observed changes in test scores cannot be considered fully independent measures, and the three students excluded from the paired comparison may have differed systematically from those included in the analysis. Studies across multiple schools and with longer interventions could examine how conceptual understanding develops over time and across different contexts. Research with greater access to digital devices could also compare individual, small-group, and whole-class use of GeoGebra. Comparative or mixed-methods studies may further examine how different forms of teacher scaffolding and student interaction contribute to the movement from dynamic exploration toward mathematical explanation and proof.

8 Conclusion

This action research examined how Grade 10 students developed conceptual understanding of circle theorems during a 15-day, GeoGebra-supported instructional sequence in a resource-constrained Nepali classroom. Within the four levels of evidence used in this study, the data show consistent gains in recognition and application of circle-theorem relationships, more uneven evidence of explanation, and only limited evidence of mathematical justification. The dynamic, projected representations gave the class a shared mathematical object to observe, question, and argue about, but the movement from a noticed invariant to a reasoned explanation depended on task design, teacher questioning, and peer discussion rather than on the software itself. The study therefore contributes a situated account of what a teacher-mediated dynamic geometry environment makes possible under conditions of limited device access, and of what remains dependent on pedagogy.

Data Availability Statement

The raw data supporting the conclusions of this article will be made available by the corresponding author, without undue reservation.

Ethics Statement

The Research Committee of Kathmandu University School of Education approved the research procedures involving human participants. The researchers obtained informed consent from all individual participants for participation in the study. Likewise, participants provided written informed consent to publish any potentially identifiable name, images, or data included in this article.

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Author contributions

All authors contributed equally from conceptualization, writing –– original draft, and writing — review & editing.

Conflicts of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Appendix A

Assessment Tasks, Marking Scheme, and Representative Items

Pre–Test and Post–Test Questionnaire

Total Marks: 10 Time: 20 minutes Class: 10

1. Explain the following terms with respect to circles:
   a) Radius (1 mark)
   Ans:_____
   b) Diameter (1 mark)
   Ans:_____
   c) Chord (1 mark)
   Ans:_____
2. Write "T" for true and "F" for False:
   a) All radii in a circle are of the same length. _____ (0.5 mark).
   b) The diameter of a circle is double of its radius. _____ (0.5 mark)
3. Multiple Choice Question (Give $\surd$ for correct answer).
   Which of the following is true? (1 mark)
   i. A tangent to a circle intersects it exactly at two points.
   ii. A chord is a line that passes through the centre of a circle.
   iii. Radius is a line segment joining the centre of a circle and a point on the circle.
   iv. A circle doesn't have any lines of symmetry.
4. Fill in the Blanks:
   a) A tangent to a circle is perpendicular to the _____ at the point of contact. (0.5 mark)
   b) The length of the longest chord of a circle is called the _____. (0.5 mark)
5. Short Answer Questions:
   a) If the radius of a circle is 5 cm, what is its diameter? (1 mark)
   Ans:_____
   b) Draw a circle, label its centre O and mark two points A and B on the circumference. Now, draw the line AB and label it as a chord. (1 mark)
   Ans:_____
6. Conceptual Question:
   Explain briefly what you understand by "congruent circles" and give one example. (1 mark)
   Ans:_____
7. Problem Solving Question:
   In a circle, if two radii form an angle of 90° at the center, what type of triangle is formed between these two radii and the chord connecting their endpoints? Explain. (1 mark)
   Ans:_____

Table 5 Scoring Notes

Scoring consideration	Marking rule
Correct response	Awarding the full mark allocated to the item.
Partially correct response	Awarding only the mark corresponding to the correctly demonstrated component where partial credit is specified.
Incorrect response	Awarding 0 marks.
Blank/no response	Awarding 0 marks.
Equivalent mathematical wording	Accepting if the mathematical meaning is correct, even when wording differs from the expected response.
Q5(b), construction/drawing	Awarding marks according to the stated components; minor drawing imperfections should not result in loss of marks if the required geometric relationships are correctly represented.
Q6, conceptual response	Accepting mathematically equivalent explanations of congruent circles.
Q7, reasoning	The identification of a right-angled triangle alone receives 0.5; the remaining 0.5 requires an appropriate explanation.

Table 6 Marking Scheme

Question	Assessment task	Expected response / criteria	Mark allocation	Total
1(a)	Define radius.	Correctly identifies a radius as a line segment joining the centre of a circle to a point on its circumference.	1	1
1(b)	Define diameter.	Correctly identifies a diameter as a chord passing through the centre of a circle, or a line segment joining two points on the circle and passing through the centre.	1	1
1(c)	Define chord.	Correctly identifies a chord as a line segment joining any two points on the circumference of a circle.	1	1
2(a)	State whether all radii of a circle have the same length.	True (T)	0.5	0.5
2(b)	State whether the diameter is twice the radius.	True (T)	0.5	0.5
3	Identify the correct statement about a circle.	(iii) Radius is a line segment joining the centre of a circle to a point on the circle.	1	1
4(a)	Complete: A tangent is perpendicular to the ... at the point of contact.	Radius	0.5	0.5
4(b)	Complete: The longest chord of a circle is called the ...	Diameter	0.5	0.5
5(a)	Find the diameter when the radius is 5 cm.	Uses $d = 2r$: $d = 2(5) = 10$cm.	1	1
5(b)	Draw and label a circle with centre O, points A and B on the circumference, and chord AB.	Circle drawn; centre labelled O; points A and B correctly marked on circumference; segment AB drawn and labelled as a chord.	1	1
6	Explain congruent circles and give an example.	0.5 mark: Correctly explains that congruent circles have the same radius (and hence the same size). 0.5 mark: Gives a valid example, such as two circles each having radius 4 cm.	$0.5 + 0.5$	1
7	Determine the type of triangle formed when two radii subtend 90° at the centre and explain.	0.5 mark: Identifies the triangle as a right-angled triangle. 0.5 mark: Explains that the two radii form a 90° angle at the centre.	$0.5 + 0.5$	1
		Total		**10**