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RESEARCH ARTICLE

Comparison of aero engine component lifing methods

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Abstract: Failure of critical engine components such as compressor, fan, and turbine disks during flight can cause the loss of the engine, aircraft, or even life. To reduce the risk of this failure during flight, different methodologies and tools have been developed to determine the safe operating life of these critical disk components. The two most widely used lifing methods, safe-life and damage tolerance, are inherently conservative, retiring all components when a predetermined operating limit is reached. Both methods retire components with theoretical useful life remaining. Additional lifing methods can be used to reduce this conservatism and extend the life of these components. Retirement for cause, developed within the United States Air Force is a lifing method that can extend the life of components by retiring a component only when there is cause to do so. Military and industry standards on lifing methodologies were reviewed. Both deterministic and probabilistic approaches to disk lifing methods are discussed as well as current tools. This paper provides a comparison of the methodologies and tools currently being used today by both the government and industry.

Keywords: gas turbine disk components, lifing methods, damage tolerance, crack propagation, inspection interval

1 Introduction

Failure of critical engine components such as compressor, fan, and turbine disks during flight can cause the loss of the engine, aircraft, or life [1]. To reduce the risk of this failure during flight, different methodologies and tools are used to determine the safe operating life of these critical components. These methodologies try to balance the potential risk of failure with the economics of component replacement. This paper provides a comparison of the methodologies and tools currently being used by both the government and industry.

It must be noted, the military and civilian agencies responsible for airworthiness requirements use differing approaches to determine the useful "life" of these critical components. The Federal Aviation Administration (FAA) [2] working with their industry partners uses design target risks (DTR) developed from their historical database and a deterministic approach to calculate a low cycle fatigue (LCF) component safe life. This deterministic method uses component analysis, testing, and safety factors to account for variability in design parameters. Alternatively, the United States Air Force (USAF) [3] uses a damage tolerance method which assumes that all materials and components have inherent flaws and thus determines the required inspection interval needed to prevent these flaws from growing to an unacceptable or catastrophic limit. USAF contractors use this method to meet airworthiness requirements. Catastrophic crashes such as occurred in Sioux City [4], Iowa led the FAA to adopt the USAF damage tolerance approach and make it an additional requirement to support the safe life limit, but not replace it. USAF has extended its damage tolerance approach to include retirement for cause (RFC) concepts whereby they could exceed the traditional safe life limit and only retire components when cracks or anomalies are found during an inspection. However, to adopt RFC as a regulatory guideline a probabilistic approach is deemed necessary to handle the variability in the design parameters. Thus, we see two principal methods have risen as the preferred approaches to estimating the safe operating life of critical engine components. First, the traditional deterministic method which uses component analysis, testing, and safety factors to account for variability in design parameters and yields a value which ensures that only 1 in 1000 parts will fail [3]. Second, the probabilistic damage tolerance approach which assumes the design variables have some distribution of values and uses simulation and statistical techniques to establish safe inspection intervals for each component. The basis for these methods will be discussed in greater detail in the following pages. There are a number of tools employed in order to exercise these approaches. The tools discussed in this paper include DARWIN, ProFES, AFGROW, and Zencrack, as well as some standard finite element codes.

2 Deterministic approach

Deterministic design has classically been used for most mechanical design, including aero engine components. With a deterministic approach, each component is analyzed to determine if it could survive what is considered to be the worst case scenario, using minimum material properties, the most critical geometry [2, 3, 22], and the assumed worse case loading, for that component. The assumed maximum stress for each component is then compared to the allowable stress for a given material with applied experience-based safety factors to determine if the component will fail [5, 12]. These safety factors are applied to account for the numerous uncertainties and product variability [18]. Zaretsky and Melis state that the deterministic approach assumes that when the strength of the material is greater than the applied stress the component will have zero probability of failure [5].

Although the deterministic design approach has the benefit of not requiring a large amount of computation, it does have its drawbacks. Zaretsky and Melis state that the deterministic method assumes that the full extent of the service conditions and the material strength is known [5]. This information is hard to obtain since materials are not homogeneous, and flight profiles and engine loading can only be estimated during engine design. Zaretsky and Melis also note components designed to have a zero probability of failure would require a structural weight and size greater than would be compatible for aircraft engines [5]. This would increase the weight and size of the engine past acceptable limits. In addition, the variability in heat treatment, loading, and operating profile, among other variables, is not necessarily factored into these calculations and that this omission can result in large variances in the component's performance, life, and reliability from those predicted using a deterministic approach [12].

3 Probabilistic approach

A deterministic approach assumes that design variables such as loading conditions, component geometry, and material properties are known values with no variation. However, experience has shown that these design variables do vary during component manufacturing [13]. This variance induces uncertainty that needs to be accounted for in component design. Chamis states that "probabilistic methods offer formal approaches to quantify those uncertainties and their subsequent effects on material behavior, on service, and on attendant reliabilities and risks [13]."

The Federal Aviation Administration Advisory Circular 33.70-2 defines probabilistic risk assessment as a fracture-mechanics based simulation procedure using statistical techniques to mathematically model and combine the influence of two or more variables to estimate the likelihood of various outcomes for a product [10].

With probabilistic approaches the mean, standard deviation, distribution, and scatter range for the variance in each design variable is determined. The purpose of these distributions is to model the variance in material properties and geometry, inherent in manufacturing, and the uncertainty of loading during component use. These distributions can most accurately be calculated by repeated measurement until sufficient data is generated to determine a distribution. In many cases this is not practical due mainly to the large number of required measurements to accurately represent each distribution. However, the scatter of the design variables can be represented by simple and well known statistical distributions and that the range of the scatter can be established from experience in practical cases.[10 In practical cases, only enough measurements to determine a representative mean and standard deviation for each variable is required. Experience can then be used to determine a representative statistical distribution and scatter range for each design variable.

With a deterministic approach component failure is calculated once using a single set of design variable values. As stated before, if the calculated stress under "worse case loading" is less than the allowable stress with an experience-based safety factor the component is expected to never fail. However, the values for the design variables are not static. Therefore, it is not possible to capture component reliability based on a single analysis using a single set of values from a distributed field of values. Each set of values in the distributed field of values has a deterministic solution. There by creating a distributed field of solutions representing all possible outcomes.

Component reliability can more accurately be determined using a probabilistic sampling approach, such as Monte Carlo Simulation. Probabilistic sampling methods analyze the deterministic solution thousands of times using values randomly selected from each variable distribution. Through this process the probability of failure (POF) can be determined for the component [26].

A probabilistic approach to design with a known closed form solution is illustrated using a simple example of the probabilistic analysis of a cantilever beam tip displacement loaded at the free end, which can be seen in the top left of Figure 1 [13]. The equation or deterministic model governing the tip displacement of the beam is shown under the loading schematic in Figure 1.

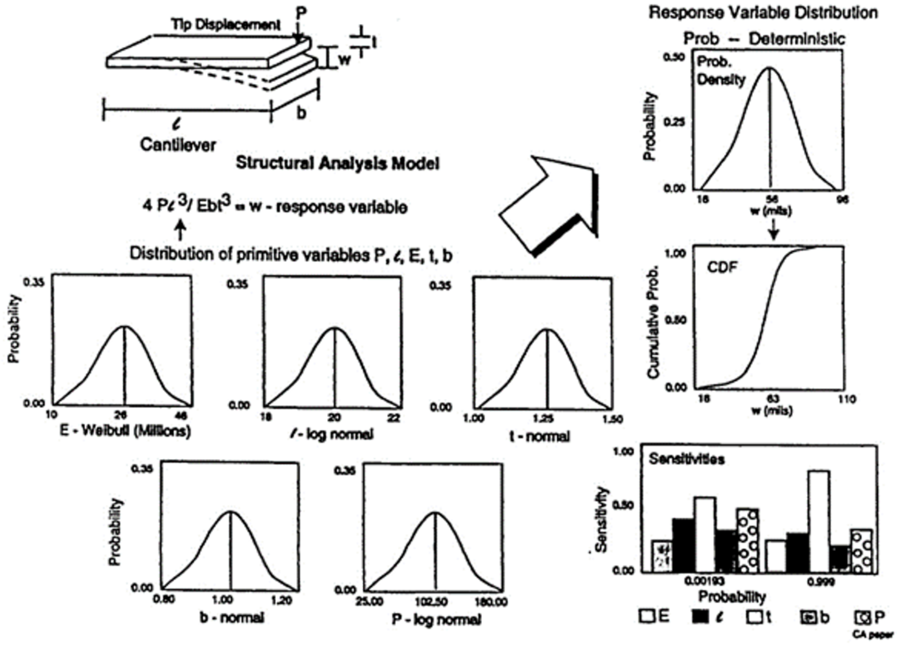

Figure 1 Probabilistic structural analysis of a cantilever beam tip displacement [13]

This equation shows the physics and parameters or design variables that govern the tip displacement of the beam. If the closed form solution was not known, or the loading or geometry was not simplistic, finite element analysis or other software can be used to determine a model response to applied loads. The design variables for this example are the load P, the length *l*, the material stiffness E, the width b, and the thickness t. These design variables fall into three generic categories: load (P), geometry (*l*, b, and t), and material (E). The task of probabilistic approaches is to account for the effects of the variable scatter on the displacement of the beam [13]. The scatter, range, and distributions used for each primitive variable can be seen in Figure 1. With the scatter, range, and distribution of each variable determined, a random value from the respective distribution is selected for each variable and the deterministic solution is determined for tip displacement of the beam. This sampling process is then repeated until sufficient data has been created to plot the probability distribution function (PDF) and cumulative distribution function (CDF) of the tip displacement of the beam [26]. The PDF and the CDF for this example can be seen in top and center right of Figure 1, respectively [13]. The probability of failure of the beam is simply the probability of the tip displacement being equal or greater than the determined failure displacement.

The benefits of probabilistic approaches are that they better account for variability without over- designing the component. Also, a by-product of this approach is the sensitivity factors identify which variables are contributing most to the uncertainty [13, 18]. Casare and Sues note this allows the analyst to determine which variables should be better controlled to improve product reliability [18]. The sensitivity factors for this example can be seen in the lower right of Figure 1.

However, probabilistic approaches can require a large amount of computation and time. A Monte Carlo Simulation can require more than 10_8 samples to converge to a solution [11]. To combat this, more complex sampling methods and advanced methods such as First Order Reliability Method (FORM), that use mathematical optimization over repeated sampling have been developed [26]. In addition, commercial probabilistic software, such as DARWIN and ProFES are available.

4 Safe-Life

Since component design with a zero probability of failure would produce an engine too heavy and too large for most applications, Zaretsky and Melis state design for a finite life with an acceptable probability of failure or risk must be considered [5]. Safe-Life is a tradition deterministic method for calculating the in-service life of rotating aircraft engine components subject to low cycle fatigue (LCF). The FAA, the US Air Force, and industry have used Safe-Life for over 50 years. Under safe-life, the in-service life to which a component can safely be used is obtained from component and material specimen tests under conditions representative of in-service loading. From this testing the component LCF life can be determined by fitting a statistical model to the fatigue results. Component life is then determined based on the likelihood of crack initiation after a given amount of engine usage. This safe-life is normally defined as the engine usage required to produce a 0.8mm surface crack in one component out of an assumed population of 1000 identical parts (1 out 1000 represents approximately three standard deviations from the mean, -3σ, see Figure 2) [7]. This crack size is considered to be detectable with a high reliability using common non-destructive inspection (NDI) techniques [8]. As an additional safety measure components are inspected using NDI at 50% of the predetermined operating limit to try to account for defects and damage which are not accounted for in the safe-life analysis.

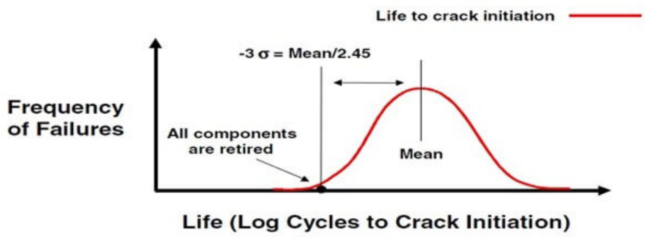

Figure 2 Graphical definition of the safe-life methodology [7]

The goal of the safe-life methodology is to ensure that all components are retired before a crack can initiate. Under safe-life all components are retired based on the likelihood that only 1 out of 1000 components will develop a detectable crack [7, 8].

The advantage of safe-life is that the maintenance requirements are able to be kept to a minimum while the time in service of components, without the need for inspection, is maximized [7]. One disadvantage is that since 999 out of 1000 components are retired with usable life remaining this lifing methodology is inherently conservative [7]. Also, if the safe-life approach was always accurate no mid-life component inspection would be required and components could be kept in service till the determined life limit was reached without the need for inspection.

Wicks, Antoniou, Slater, and Hou believe the biggest disadvantage of using safe-life is that it is unable to account for "rogue" flaws, such as surface damage due to abusive machining or abnormal material microstructures [8]. Since safe-life is based on limited component and material testing these flaws are not accounted for in the original safe-life calculation due to their low occurrence rate. Therefore, the safe-life method cannot accurately predict the life of these components. Ejaz, Salam, Tauqir present a failure report on a recent fighter aircraft crash that was caused by the uncontrolled failure of a 9^{th} stage compressor disk during takeoff [1]. They conclude that this uncontrolled failure was caused by fatigue cracks that initiated at "deep machining marks" on the surface of the disk, as seen in Figure 3 [1]. In these cases, safe-life can go from overly conservative to over estimating components safe operating life [8]. Safe-Life alone represents only a partial view of the whole process that should be taken into account for determining the useable life of components [8].

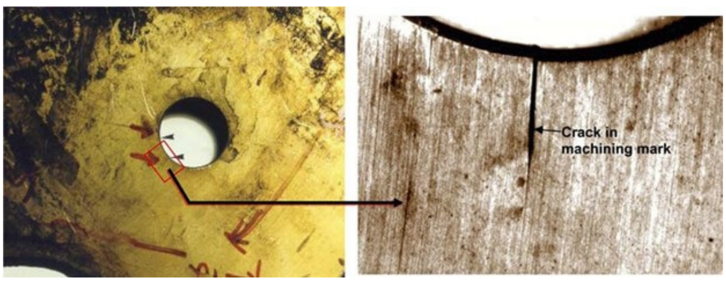

Figure 3 Failed compressor disk caused by crack initiation at deep machining marks [1]

5 Damage tolerance

In 1989, United Airline Fligt 232 in Sioux City, Iowa experienced a disk burst caused when a disk ruptured due to a crack that initiated from a titanium material defect, known as hard alpha [14]. The Federal Aviation Administration (FAA) and the Rotor Integrity Subcommittee (RISC) of the Aerospace Industries Association (AIA) proposed a probabilistic damage tolerance approach to account for inherent material structure and manufacturing defects. This recommendation led to the development of a new FAA advisory circular and the Turbine Rotor Material Design (TRMD) research program.

The FAA Advisory Circular 33.70-2 [2] states that "in the absence of anomalies, the safe-life methodology provides a structured process for the design and life management of high-energy rotors" and that "undetectable manufacturing, material melt-related, service-induced anomalies, therefore, represent a departure from the assumed nominal conditions [10]." Since the safe-life approach is unable to account for these flaws the damage tolerance approach was developed as a supplement to the safe-life approach [10].

Under the damage tolerance approach, components are assumed to contain inherent material structure, crack-like manufacturing, or service induced defects in fracture critical areas that give rise to crack propagation during service [7, 10]. These crack-like defects are assumed to be of a size just below the detection limit of the non-destructive inspection techniques used for component inspection [14]. This approach further assumes components are able to continue safe operation as these cracks grow during usage [6]. In addition, these cracks are assumed to grow in a manner that can be predicted using linear-elastic fracture mechanics (LEFM) slowly enough to allow their detection through scheduled component inspections [6]. These assumptions must all be substantiated by material and component testing. The inspection schedule is established using analysis and verifying testing to ensure that cracks will not grow beyond a set limit, known as dysfunction or critical crack length, between inspections. The critical crack length is defined as the crack size at which the risk of rapid or unstable crack growth reaches an unacceptable level and is determined using assumed crack geometry and the crack tip stress intensity factor [6].

The key to the damage tolerance approach is determining the safety limit and the inspection time interval. The safety limit is the calculated time it takes a hypothetical crack to grow from a size just below the NDI detection limit to the critical crack length. The safety limit is then divided by a safety factor, which is normally two, to determine the inspection time interval or safe inspection interval (SII) [14]. Components are then inspected each time the SII is reached using NDI methods. The damage tolerance approach is described schematically in Figure 4.

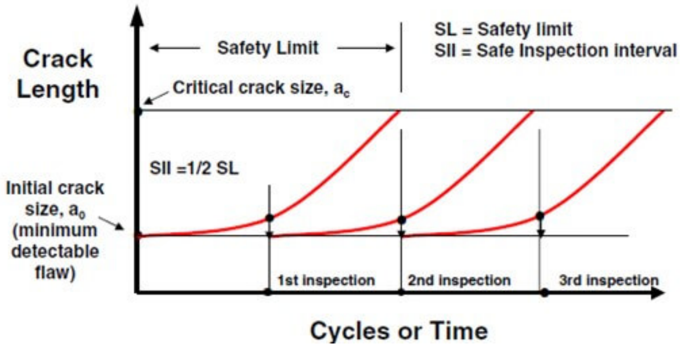

Figure 4 Graphical definition of the damage tolerance lifing methodology [7]

The damage tolerance and the safe-life approaches are similar in that both predetermine a safe operating life and retire all components once the predetermined limit is reached. The advantages of the damage tolerance approach are that it ensures that no cracks will grow beyond a critical crack length between inspection intervals and it accounts for flaws inherent in the material and caused by manufacturing. However, Immarigeon note that this approach can be more costly to implement than the safe-life approach, since it requires an elaborate NDI infrastructure to handle the increased inspection requirements and increases the handling on the components [6]. Increasing component handling increases the probability of induced component defects.

5.1 Damage tolerance analysis tools

There are several commercially available programs to help with implementing a probabilistic damage tolerance approach. These programs allow the analyst to use predefined codes and probabilistic methods to determine component reliability. The two such programs are DARWIN and ProFES.

Through the TRMD research program, the Southwest Research Institute created the probabilistic damage tolerance computer program known as DARWIN (Design Assessment of

Reliability With INspection) [17]. First released in 1997, DARWIN uses finite element (FE) stress analysis and combines it with fracture mechanics analysis, probability of detection (POD) curves , and probabilistic analysis to determine the probability of rotor failure [15]. The program calculates the probability of failure (POF) as a function of flight cycles, taking into account random defect occurrence and location, random inspection schedules, and other random variables. DARWIN calculates the POF caused titanium hard alpha or general inherent material defect occurrences by using a zone-based methodology where by the two-dimensional section of the component is divided into sections called zones, as seen Figure 5 [15].

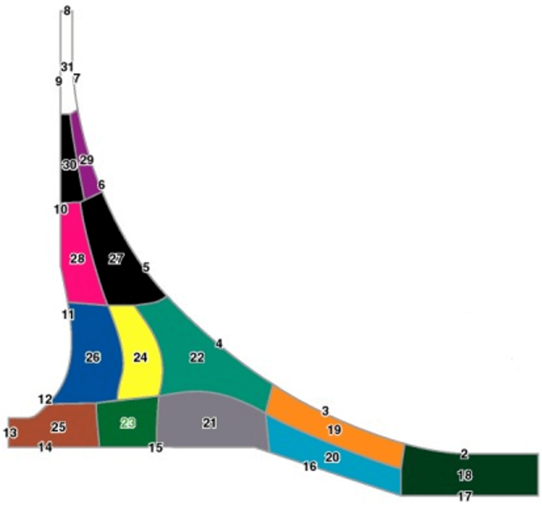

Figure 5 DARWIN 2D rotator cross section zone discretized component model [17]

DARWIN defines a zone as a section of material such that all locations in the zone have a generally uniform stress state, the same fatigue crack growth properties, inspection schedule, probability of detection, and anomaly distribution [4]. This means that the computed risk for any subsection of a single zone will be approximately the same; and therefore, the risk is assumed constant for a defect located anywhere in that zone [4]. With the zones defined, the POF can then be determined for each zone using probabilistic sampling, which then can be used to determine the POF of the entire rotor.[12]

A zone-based approach ensures that probabilistic sampling is performed in each zone. Millwater states this allows the sampling to be tailored based on knowledge of likely high risk regions of the component, which can lead to an efficient probabilistic approach, reducing needed computation [4]. However, since the zones are user defined the accuracy of the results are dependent on the user's discretization of the zones (size and number of zones) [4]. Recognizing this issue, a zone refinement tool was added to DARWIN in 2002 to help users with zone discretization [15].

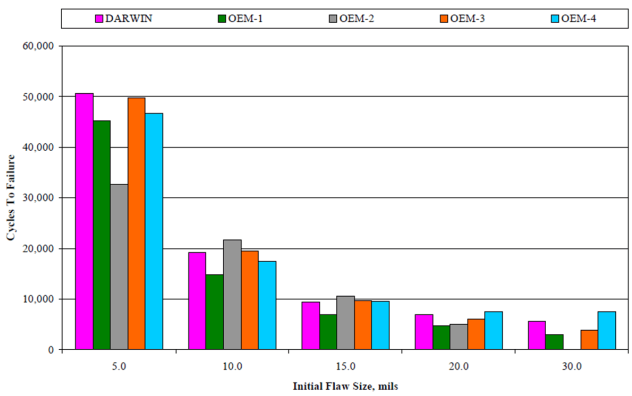

Figure 6 DARWIN comparison to OEM proprietary POF assessment codes [15]

To confirm the accuracy of DARWIN, it was independently evaluated by original equipment aircraft manufacturer members of the AIA Rotor Integrity Subcommittee. These original equipment manufacturers (OEM's) compared DARWIN one-on-one with their own proprietary POF assessment codes to predict the low-cycle fatigue crack propagation lives for different location in a hypothetical Ti-6A1-4V rotating ring [15]. The results of these evaluations can be seen in Figure 6. The figure shows that DARWIN is bracketed by the results from the OEM codes on four of the five evaluated flaw sizes.

The FAA has endorsed DARWIN as an acceptable tool to conduct risk analyses for certification of new titanium rotor designs in compliance with Advisory Circular 33.14 [15] and its replacement Advisory Circular 33.70. A calibration test problem is provided in Advisory Circular (AC) 33.70-2 for the probabilistic risk assessment of manufacturing-induced anomalies in circular holes [10].

AC 33.70-2 includes the necessary information to determine the probability of failure of a rotating Ti-6Al-4V ring with 40, ½ inch diameter, bolt holes both with and without inspection. The disk is rotated at a maximum speed of 5,700 rpm at room temperature with an external pressure load P of 4.786 ksi applied at the outer diameter to "simulate blade loading [10]." The geometry and loading can be seen in Figure 7 and 8.

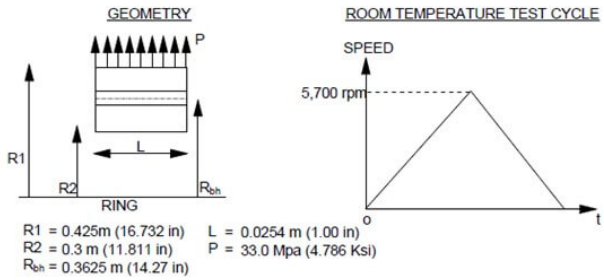

Figure 7 AC 33.70-2 calibration test provided geometry and loading [10]

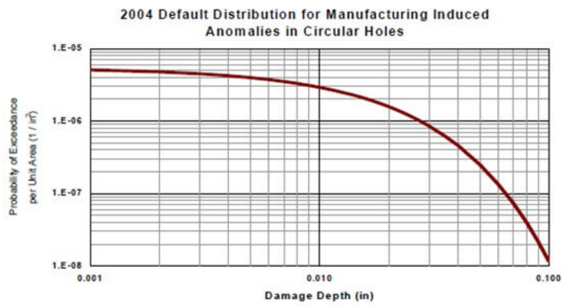

Figure 8 AC 33.70-2 calibration test provided distribution for manufacturing induced anomalies in circular holes [10]

The probability of detection (POD) curves for eddy current of finished machined surfaces is also provided in the paper and can be seen in Figure 9. The AC 33.70-2 specifies that the line for the "reject at one half the calibration notch threshold response and above" should be used for the stated problem. This is represented by the doted in line in Figure 10. Two in-service eddy current inspections are to be preformed: one at 4,000 cycles; and the second at 8,000 cycles. At each of these two inspections an independent 90 percent of the components are to be inspected.

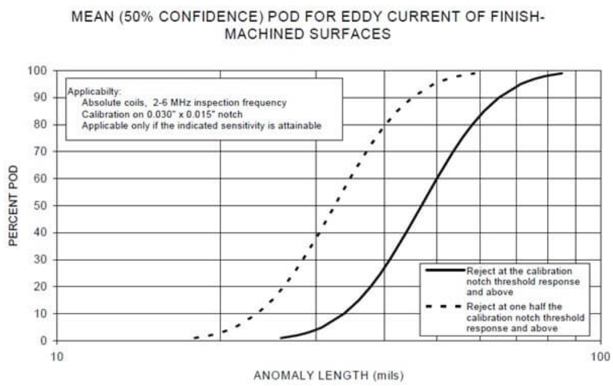

Figure 9 AC 33.70-2 2 calibration test provided POD for eddy current of finished machined surfaces [10]

The provided material properties for Ti-6Al-4V can be seen in Table 1.

Table 1 Ti-6Al-4V Material Properties

Properties	Value
Young's Modulus (E)	17.4E3 ksi
Density	0.161 lb/in^3
Poisson's Ratio	0.361
Yield Strength	121 ksi
Ultimate Tensile Strength	132 ksi
Fracture Toughness	58.7 ksi$\sqrt{\text{in}}$

The crack propagation data is also provided. K threshold is given as 0 ksi$\sqrt{\text{in}}$ and two crack growth properties for two R-ratios are given in the form of the Paris equation:

$$\text{For R} = 0: \quad \text{da/dN} = 5.248E - 11(\Delta K)^{3.87} \tag{1}$$

$$\text{For R} = -1: \quad \text{da/dN} = 7.2684E - 12(\Delta K)^{3.87} \tag{2}$$

AC 33.70-2 notes, the hole is to be subdivided into four crack locations as seen in Figure 11. AC 33.70-2 states, the "relative risk or probability of failure (POF) is calculated for each location. Results for the highest location are multiplied by the number of holes and by the surface area per hole to arrive at the total hole set POF or relative risk [10]." The AC states that the POF can be calculated by either an "integrated probabilistic method or a 'Monte Carlo' method [10]." The AC also notes that if a Monte Carlo approach is used then the number of samples needs to be at least two orders of magnitude greater than the computed risk [10].

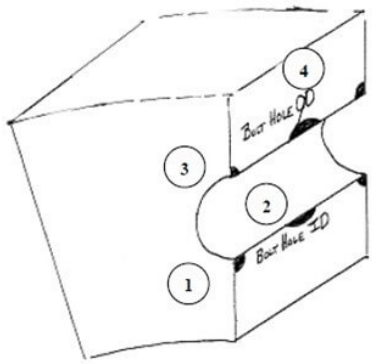

Figure 10 Created AC 33.70-2 POD curve for eddy current of finished machined surfaces

The paper notes several manufacturers have preformed this problem. The statistical analysis of the results in terms of events per service life of 20,000 cycles can be seen in Table 2, where "m" is the mean and "s" is the sample standard deviation. These two ranges define intervals that are center on the mean value and cover 90 percent of the population assuming a log-normal distribution.

Table 2 Statistical analysis of events per service life of 20,000 cycles [10]

Failure Risk Events/Service Life	Mean Value (m)	m - 1.65 s	m + 1.65 s
Without in-service inspection	2.88E-04	2.58E-04	3.22E-04
With an in-service inspection	1.28E-04	7.29E-05	2.25E-04

Although the goal of the Advisory Circular 33.70-2 calibration test case is to provide all the needed information for the calibration test case, two essential items were discovered missing. Firstly, although tabular data is provided for the anomaly distribution, no such tabular data is included for the POD curve. This forces the user to use primitive methods to match the curve which could lead to unnecessary uncertainty in the results. The POD that was created and used can be seen in Figure 10 with the tabular data in Table 3. Secondly, a stress-strain model is not specified in the AC. This is required material information in DARWIN and must now be assumed by the user.

A terminology issue was also noted between the AC 33.70-2 and DARWIN. The anomaly distribution in the AC 33.70-2 is given in terms of the probability of exceedance per unit area, where the anomaly distribution format for DARWIN is in terms of number of exceedances per unit area. It is believed that these terms are interchangeable and so the given anomaly distribution was used without modification.

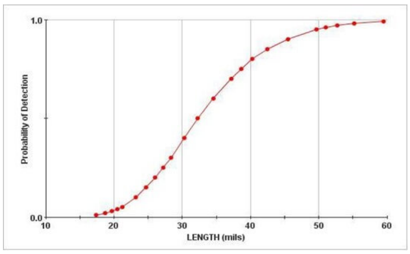

Figure 11 AC 33.70-2 2 calibration test crack locations defined [10]

Table 3 POD with different length

Length (mil)	POD
17.39045	0.01
18.71669	0.02
19.68953	0.03
20.50476	0.04
21.22864	0.05
23.20661	0.10
24.71664	0.15
26.02601	0.20
27.22793	0.25
28.36819	0.30
30.31526	0.40
32.26153	0.50
34.57147	0.60
37.19241	0.70
38.65249	0.75
40.25581	0.80
42.46918	0.85
45.5	0.90
49.65504	0.95
51.03531	0.96
52.71739	0.97
55.19658	0.98
59.49937	0.99

The Advisory Circular 33.70-2 calibration test case was performed using DARWIN. The stress at maximum loading was determined using ABAQUS 6.8-1. As can be seen in Figure 12 and 13, the maximum stress is located at the inner and out diameter of the inner surface of the bolt hole in the hoop direction. This is to be expected as these areas coincide with the crack locations specified in the AC. The maximum principle stress was determined to be 126.1 ksi (869MPa).

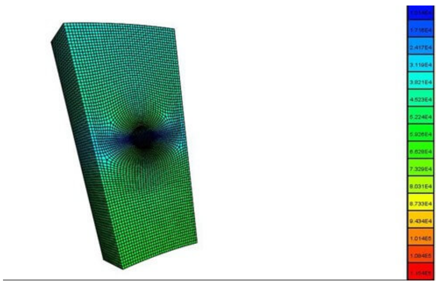

Figure 12 AC 33.70-2 2 calibration test maximum principle stress analysis using ABAQUS

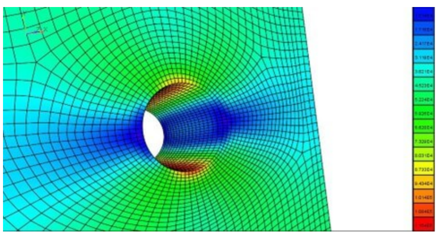

Figure 13 AC 33.70-2 2 calibration test maximum principle stress analysis using ABAQUS

Before the stress and geometry can be entered into DARWIN, the finite element results file must be converted into the two neutral files. These two neutral files: the "uif" containing only nodal locations and the "uof" containing only stress and temperature information are the required input format for DARWIN. This conversion is accomplished through the use of the DARWIN feature FE2NEU. After creating the neutral files, the stress profile and geometry along with all the provided information in AC 33.70-2 were loaded into DARWIN.

Since a stress-strain model was not specified the Ramberg-Osgood relationship was used:

The β strain hardening exponent for the material, and α^{σ_o} is the yield offset. For this problem β was set to 20 and α was rounded to 0.28/6, this information was used from "Surface Damage Tolerance Analysis of a Gas Turbine Rotor" by Jameel [39]. In order to use this information in DARWIN the Ramberg-Osgood equation was used in a specific form.

Our calculation revealed, strain to be 0.00695.

Since AC states the POF for the ring is determined by multiplying the crack location with the highest individual POF by the surface area of the 40 holes a sensitivity analysis must be preformed first. The crack location with the highest sensitivity contributes the most to the POF and therefore, has the highest individual POF. This is accomplished by performing the POF analysis in DARWIN with all four crack locations present. A Monte Carlo sampling method with life approximation function (LAF) was used for all presented analyses. For the sensitivity analysis 50,000 samples per zone were used and 100,000 samples per zone were used for all POF analysis. It is also important to note that each sample is seen as an individual disk in a fleet of disks equal to the number of samples. The LAF is an option in DARWIN that creates an approximate response surface for the component life. The DARWIN user's manual states that this response surface is based on a limited number of calls to the fatigue crack growth code and this option dramatically reduces the required computation, while maintaining sufficient accuracy [40]. The results from the sensitivity analysis can be seen in Figure 14 and Table 4.

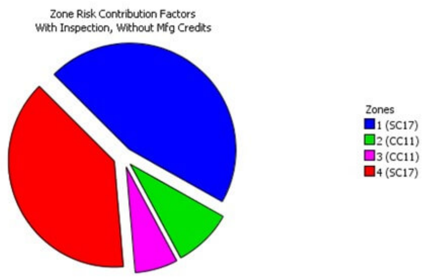

Figure 14 AC 33.70-2 calibration test crack location sensitive analysis using DARWIN

Table 4 AC 33.70-2 crack location sensitive analysis

Crack Location	Zone	Risk % With Inspection
1	2	8.94
2	1	45.69
3	3	6.46
4	4	38.90

From this analysis it can be seen that crack location 2 or zone 1, the center bolt hole crack at the inner surface, has the highest sensitivity at 45.69%. The plate geometry, stress, and crack location (white dot) for zone 1 can be seen in Figure 15. Since the AC specified that the POF for the ring is to be based on only the crack location with the highest sensitivity, crack locations 1, 3, and 4 are removed. This is accomplished in DARWIN by removing the zones associated with these crack locations: zones 2, 3, and 4.

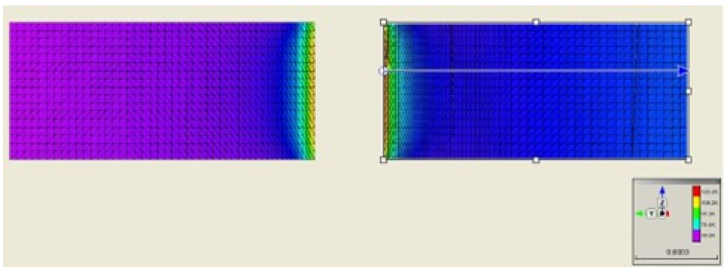

Figure 15 AC 33.70-2 calibration test DARWIN zone 1 definition

With only zone 1 remaining the total feature area (surface area of the bolt hole multiplied number of holes in the feature) must now be taken into account. However, the POF of zone 1 will not need to be manually multiplied by the total feature area as noted in the AC. The surface area and number of holes of the feature can be entered in the zoned definition in DARWIN and taken into account in the POF analysis. This can be seen in Figure 16 and 17, where the area of 0.7854 square inches and 80 features, represents 80 half bolt holes. With total feature area entered DARWIN now calculates the POF of the entire ring feature and not the singular crack location. The POF with inspection can be seen in Figure 18 and the results from this analysis can be seen in Table 5.

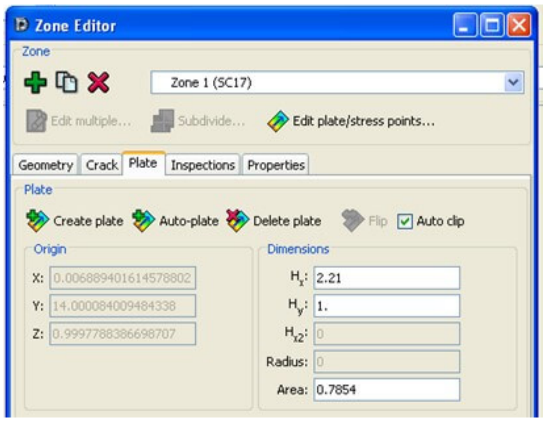

Figure 16 AC 33.70-2 calibration test half bolt hole area defined in DARWIN zone 1 definition

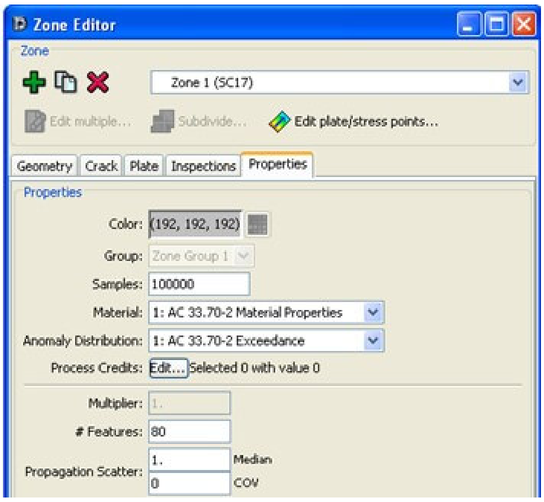

Figure 17 AC 33.70-2 calibration test number of half holes in the feature defined in DARWIN zone 1 definition

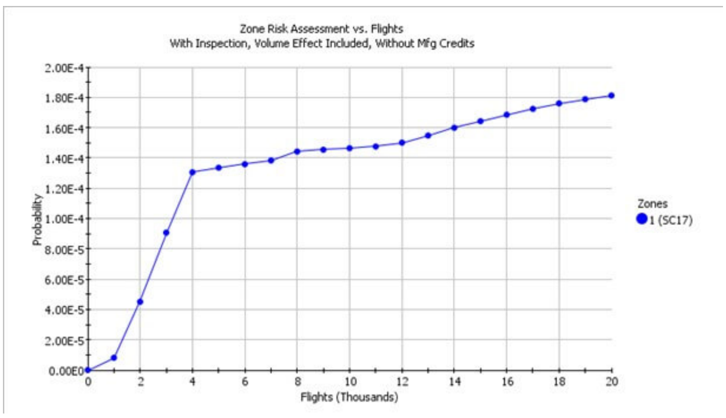

Figure 18 AC 33.70-2 Probability of Fracture analysis using DARWIN

Table 5 AC 33.70-2 Probability of Facture at 20,000 Cycles Using DARWIN

Without Inspection	With Inspection
2.886E-4	1.81E-5

Both POFs for the “with” and “without” inspection are within the acceptable ranges given in the AC 33.70-2. The POF for the zone 1 without inspection at 20,000 or the end of service life is 2.886E-4 which is only approximately 0.2 percent above the mean value for the given results.

The results for the POF for zone 1 with inspectionat the end of service are 1.81E- 4 which is approximately 41.4 percent above the mean value for the given results. Although only the POF of only zone 1 was calculated this can be viewed as the POF of the ring since the sensitive analysis shows that the failure of this ring is governed by this crack location.

Although the percent increase above the mean value from without inspection to with inspection seems like a large increase and could point to an error in the data, this should be expected. The acceptable results range for the POF without inspection is only +11.8% to -10.4%, where as the acceptable results range for the POF with inspection is +75.8% to -43.0%. The increased results range for the POF with inspection shows that there was a greater range of results by the manufacturers that preformed this problem, from which the given results are based. This is not surprising since there is higher uncertainty associated with how the impacts of inspections on the POF are calculated by the various manufacturers.

AC 33.70-2 test case uses a simplistic single load step from zero to maximum load. This type of loading represents a main cycle corresponding to the maximum stress range [42]. The effect of sub-cycles, defined as stress ranges between zero and the maximum stress range, are not taken into account in this type of loading. AA ten Have states this, “illustrates the material evaluation for disc usage is principally based on constant amplitude testing. The effect of sub-cycles, e.g. the number and magnitude of sub-cycles and their sequence, and dwell effects are not incorporated in this evaluation.” They concluded that fatigue testing should be conducted under a realistic, operational loading pattern so that relevant material fatigue data can be obtained [42].

In 1982, the National Aerospace Laboratory of the Netherlands (NLR) started a program known as TURBISTAN, to develop realistic loading sequences for fighter aircraft. They determined that loading sequences could be easily derived from the rotations per minute (RPM) sequences of fighter engines. Then from these sequences a common, representative load sequence standard could be created, allowing fatigue data comparison between different institutions [42].

The TURBISTAN group obtained 31 RPM representative sequences or missions from five different fighter aircraft engines. These 31 missions were then categorized by their loading into one of four mission types: air combat, transition or training, ground attack, and navigation. This information can be seen in Table 6 [42].

Table 6 TURBISTAN mission categorization

Mission Type	Engine					
	J79	RB199	J85	Larzac	M53	Sum
Air Combat	2	1	2	-	2	7
Transition	2	1	1	1	-	5
Ground Attack	2	2	1	4	-	9
Navigation	2	1	-	6	1	10
						Total: 31

It was further determined that in order to “most adequately represent the general structure of operational flights” that typical flight characteristics needed to be incorporated in the standard as much as possible [42]. Therefore, the missions were broken down into a list of nine “major flight events:” ground handling (1), departure (2), cruise (3), maneuvering-low (4), maneuvering-high (5), landing (6), taxi (7), engine checks (8), and thrust reversals (9).

Each of the nine major flight events were then broken down into three major blocks as defined by when the events occur within the mission. The initial block contains the events ground handling, engine checks, and departure, with each event happening only once and in that order. The centre block contains the in-flight events cruise, maneuvering-low, and maneuvering-high. Each of these three events appears in a random order for an arbitrary number of times with varying durations [42]. Lastly, the final block contains the events landing, thrust reversals, and taxi, with each appearing once in that order [42]. The general flight structure can be seen in Figure 19.

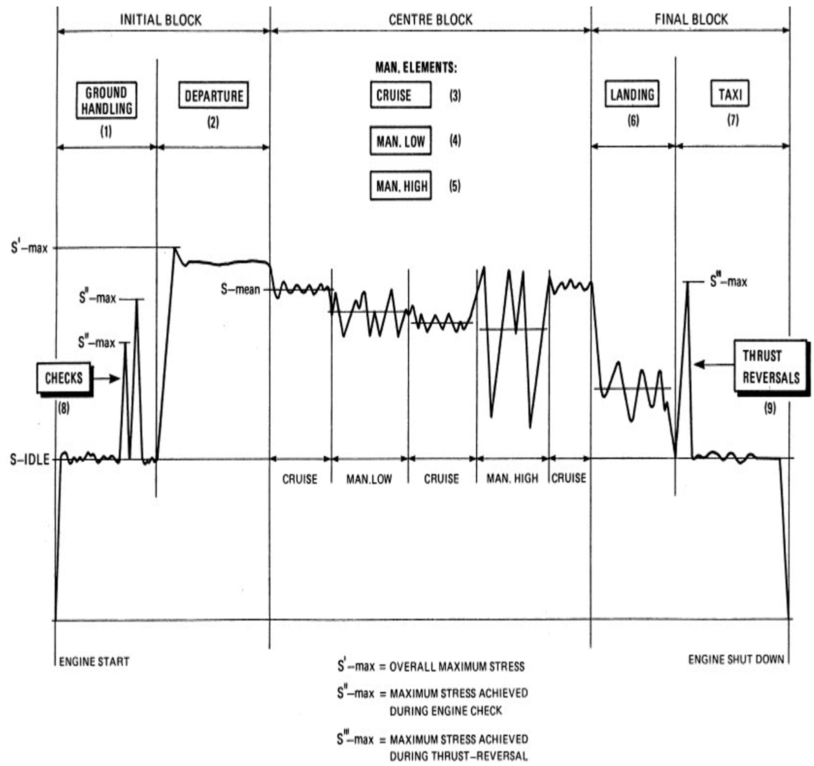

Figure 19 TURBISTAN general flight structure containing the nine major flight events and three flight blocks [42]

These events were determined to occurred in either a deterministic or stochastic manner. Stochastic flight events are characterized by a variable amplitude loading superimposed on a constant mean stress. Deterministic flight events are characterized by other parameters, such as a single load excursion with a fixed magnitude [42]. Flight events ground handling, cruise, maneuvering-low, maneuvering-high, landing, and taxi were determined to be stochastic events. This is graphically described in Figure 20. It is important to note that the maximum load is reached as least once in each mission during the departure flight event [42].

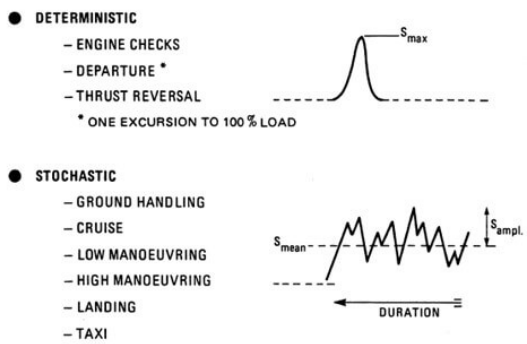

Figure 20 TURBISTAN mission event deterministic stochastic categorization [42]

With the flights and flight events categorized they were used as building blocks to create a TURBISTAN standard loading sequence of 100 flights with a mission type distribution or mission mix that can be seen in Table 7. This mission mix was considered to be representative of the average annual fighter aircraft usage [42]. This process was well documented in reference 42.

Actual flight or loading profiles can be entered in DARWIN by applying a series of scaling factors to the uploaded maximum stress and temperature. The AC 33.70-2 test case was analyzed using the 15 navigation missions of Cold TURBISTAN and given load maximum load case. The Cold TURBISTAN loading standard is defined for constant temperatures which is consistent with the test case definition which omits temperature loading [10, 42]. Since TURBISTAN is defined for a fighter engine not all of the mission types defined in TURBISTAN would be characteristic of FAA passenger planes. For this reason, only the navigation missions, which

Table 7 TURBISTAN mission mix

Mission Type	Percentage
Air Combat	20%
Transition	10%
Ground Attack	55%
Navigation	15%

are characterized by centre blocks with long cruises and few throttle changes were used [42]. Four centre block flight event sequences for TURBISTAN navigation missions can be seen in Figure 21 and Table 8 represent the missions of the passenger planes of the FAA. Consequently, the TURBISTAN missions that were used contained very few main cycles. Since the previous analysis loading profile contained a single main cycle it is not surprising such a small change in the POF was noted.

Despite DARWIN's level of industry support, there are still skeptics of DARWIN that are willing to point out possible issues with the program. Tong states that it is unclear from the available documentation as to whether variation of the random variables at each of the levels.

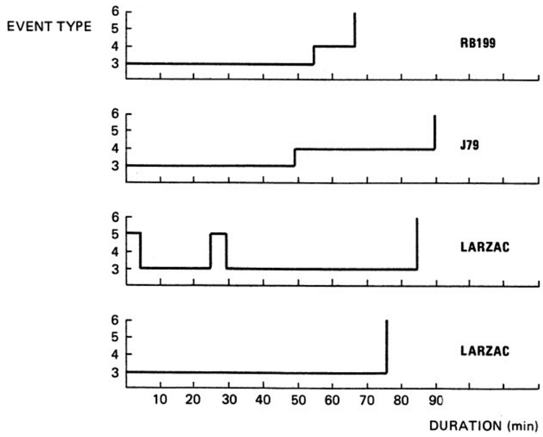

Figure 21 TURBISTAN Navigation Centre Block Flight Event Sequencing [42]

Table 8 AC 33.70-2 probability of facture at 20,000 cycles with cold TURBISTAN loading

Without Inspection	With Inspection
2.90E-4	1.84E-5

In this case using a more realistic loading profile has changed the POF of the ring very little. The POF "with" and "without" inspection using navigation missions from Cold TURBISTAN changed approximately 0.5% and 1.7%, respectively. This is for two reasons: the navigation missions contain mainly long cruises and POF of this problem is governed by main cycles or cycles with large amplitude changes. The navigation missions were chosen specifically because their long cruises made up of sub-cycles more accurately represent the missions of the passenger planes of the FAA. Consequently, the TURBISTAN missions that were used contained very few main cycles. Since the previous analysis loading profile contained a single main cycle it is not surprising such a small change in the POF was noted.

Despite DARWIN's level of industry support, there are still skeptics of DARWIN that are willing to point out possible issues with the program. Tong states that it is unclear from the available documentation as to whether variation of the random variables at each of the levels such as location, zone, or disk-related have been correctly accounted for and whether the random variables are truly assumed random for each zone [6]. They contend that if the variables are not truly random the program could produce an unrealistic result. They also point out that DARWIN viewing inspection intervals as random variables for each zone, is unrealistic since the entire disk is inspected at one time [16].

Another available computer-based probabilistic damage tolerance program is ProFES. ProFES is a program built on the concept that if the program is easy to use more people will use it.

Probabilistic tools can be complicated to use and can require extensive training to be used to their full potential. ProFES is designed so that very little training should be required for anyone with previous experience with modern deterministic finite element tools [8]. Also, ProFES has a graphical user interface (GUI) and a graphical three dimensional environment similar to modern finite element tools. ProFES allows the analyst to determine which variables are to be seen as random variables with distributions [18]. The program has several different probabilistic methods built in, including Monte Carlo Simulation and advanced methods such FORM [18]. These factors allow the user to have more control over the analysis. In addition, ProFES is able to handle more than just the design and reliability certification of titanium rotors. However, ProFES is not as widely used or have the level of industry support that DRAWIN does. Nor has it been certified by the FAA as an acceptable tool to conduct risk analyses for certification of new titanium rotor designs.

6 Retirement for cause

The safe-life method in conjunction with the damage tolerance approach, which can account for "rogue" flaws has historically provided a safe and economical approach to component lifing.

However, under both safe-life and damage tolerance, component life is predetermined and components are retired when the life limit is reached regardless of whether a life-limiting crack is found [6]. Predetermined component life can be conservative as components are retired with hypothetical usable life remaining. Vukelich stated in 2001 that a majority of the rotator components in the United States Air Force's fleet were nearing retirement based on these two approaches and with rising costs and an aging fleet additional approaches need to be reviewed to extend the life of these components [9].

One such approach is retirement for cause (RFC). This is of an extension of the damage tolerance approach since damage tolerance is normally used to determine the safe inspection interval for RFC method. However, unlike damage tolerance under RFC if no crack is detected during inspection, components are placed back in service and only retired once a crack is detected [6]. This method is graphically described in Figure 22. This allows the operator to reduce the conservatism and expand the useful life of these components.

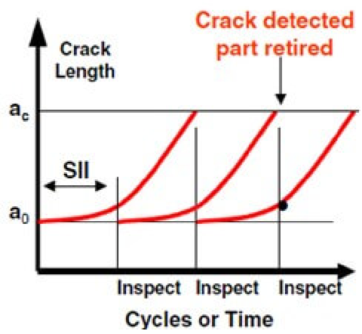

Figure 22 Graphical definition of the retirement for cause methodology [7]

In 2001 Vukelich stated that the cost of disk replacements based on LCF from 2000 to 2010 could reach close to $300 million for the US Air Force alone (see Figure 23) and that this cost could be reduced through the implementation of current RFC technology [9]. The cost savings of implementing current RFC technology based on the number of disk generations seen in Figure 23 could be as much as $100 million over the 10 year period as seen in Figure 24. A small investment in RFC technology could "more than double" the cost savings.[6]

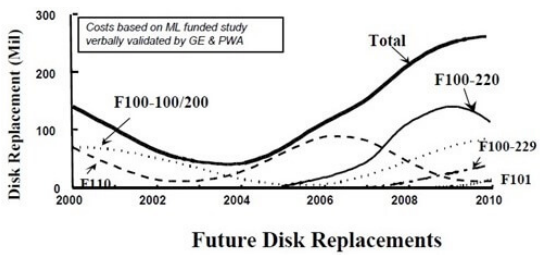

Figure 23 Projected disk component replacement for the US Air Force [9]

The disadvantage of the RFC lifing method is that as components are kept in service longer the inherent risk of failure during operation increases [9]. Since components are not retired till a life limiting crack is found, components are in operation with a yet undiscovered crack. This

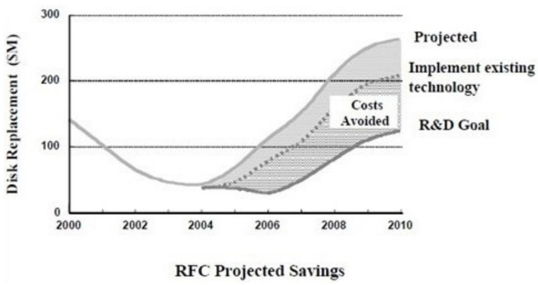

Figure 24 Projected RFC component replacement saving for US Air Force [9]

fact alone deters some from implementing this lifing method. However, Vukelich contends that it is possible to maintain the same risk as before by reducing the inspection interval.[6 Reducing the time components are in service with undiscovered cracks, reducing the chance of failure during operation.

It is also important to note that once the minimum life is reached, the inspection requirements are increased. To extend component life and maintain the same level of risk more than just the fracture critical areas must be inspected, using more than just surface inspection techniques [9]. An example is given where a high pressure turbine disk has a LCF life of 9,000 TACs. The disk was inspected at 4,500 TACs and returned to service. At 9,000 TACs the disk would be retired under other approaches, but under RFC it would be inspected and returned to service [9]. However, Vukelich notes that the entire disk must now be looked at analytically [9]. If the bolt hole area has an inspection interval of 10,000 TACs this area would now have to be inspected before the disk could return to service [9]. This is just one example of additional inspections that would now need to be conducted to extend components past their LCF life. In order for RFC to be successfully component inspection procedures for inspections past the pre-determined life must be determined and put in place before RFC is implemented.

The ability to reliably detect cracks using current NDI is also a factor that must be considered in implementing RFC. Goswami states “the successful implementation of the RFC or damage tolerant based life estimation philosophy depends largely on the sensitivity of the NDI method employed to detect the flaw crack [25].” Cracks missed during component inspection increase the risk of uncontained failure. Therefore, it is very important from a design perspective that the probability of detection (POD) for each NDI be determined [25]. A probability of detection example curve for the NDI methods Liquid Penetrant Inspection (LPI), X-Ray Radiography Inspection (XRI), and Eddy Current Inspection (ECI) can be seen in Figure 25.

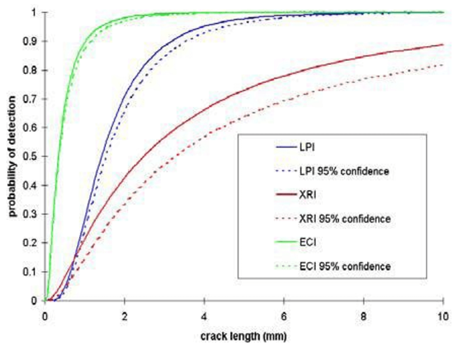

Figure 25 Sample POD curves for common NDI techniques [9]

Probabilistic approaches can be used to determine the probability of missing a crack during NDI inspection failing during operation. This analysis was conducted using LPI by Immerigeon [6]. A log-normal analysis was used for the fatigue crack growth rates (FGGR) and uncertainties associated with LPI. The author contends that the log-normal distribution models the effect of worst possible scatter in the FGGR [6]. The probabilistic analysis concluded that there was 0.1% probability of a missed crack reaching a critical crack size in 26,000 cycles as seen in Figure 26 [6]. If the worst case scenario of 5 LCF cycles per hour is assumed, the 0.1% probability of failure translates into 5000 hours of engine usage [6]. The author concludes that since the engine in the analysis is inspected every 1800 hours is not surprising that no engines have failed during operation.

Another factor in implementing RFC is increased cost of component inspection. Tschirne and Holzbecher note the cost of removal, disassembly, repair, and reassembly can be more than ten

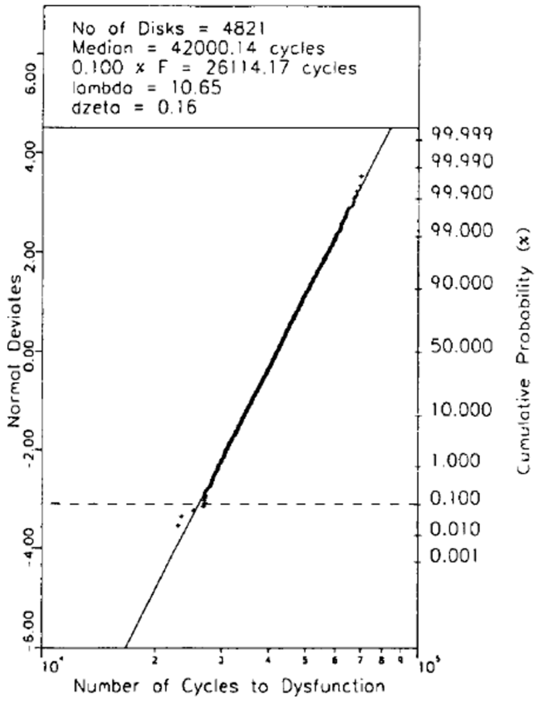

Figure 26 Probability of a missed crack reaching a critical crack size using LPI [6]

times that of a single new component [19]. In order to use RFC efficiently, consideration must be taken to balance the value of the unused life of the components with the cost of inspection.[17 Vukelich contends that "'Cause' for retirement can be based on inspection findings, unacceptable risk, or even economics [9]." An example is given where a rotor stage has been extended 2 intervals and on the 3rd interval half of the components are rejected due to inspection findings. Vukelich states that if the cost of the inspections is now equal to or greater than the cost of replacement, it would be best to retire all the disks after the 2nd interval for economic reasons [9]. They concluded that the value of 'wasted' component life and the cost for engine removal and repair have to be balanced and that each time a component becomes available, a decision must be made whether to reuse the individual item or retire it due to its remaining life being too low [9]. In order for RFC to be used to its full potential, cause for retirement must be extended to consider component availability, discovery of life limiting defects found using NDI, and economic reasons.

7 Other tools

In addition to probabilistic tools, there are other commercially available deterministic tools that use fracture mechanics to determine crack growth rates and cycles to failure of a given crack and crack geometry. Two such programs discussed in this paper are AFGROW and Zencrack.

AFGROW is a fatigue crack growth prediction program that was developed by the United States Air Force. This program uses liner elastic fracture mechanics (LEFM) to determine the crack growth rate and cycles to failure of a two dimensional cross section with a given crack [20]. The program includes a library of predefined two dimensional cross sections and material properties and the ability to define your own. The program has several built in crack growth rate models including the Walker and Forman equations and allowing for tabular inputs. AFGROW is also able to account for both crack closure and residual stress effects in the material [20]. However, the program requires known or assumed crack location and geometry. It should be noted this program was not created as a rotor design tool since it is not always clear where cracks are going to initiate and rotors are removed from service if a crack is discovered.

As an illustrative example, the cycles to failure of a titanium plate with a center hole which is loaded in cyclic tension was calculated using AFGROW. A classic model with a surface crack at a hole with a crack size of 0.001x0.001 inches was used. This information as an AFGROW input can be seen along with the overall dimensions for the plate in Figure 27. AFGROW can also account for beam bending and bearing bolt hole loads. However, when multiple loading types are used the individual load types must be normalized by the maximum load.

The AFGROW loading spectrum provides a means to enter the stress or load of the component

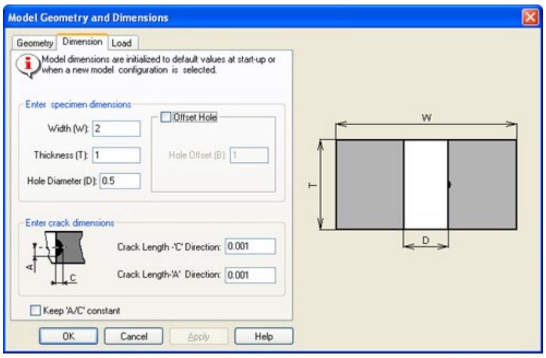

Figure 27 AFGROW titanium plate example model geometry and dimensions [6]

or structure in question. The loading spectrum can be created in a text file format or created within the program. The AFGROW spectrum input window with stress multiplication factor (SMF) can be seen in Figure 28. The SMF a scaling factor that is multiplied by each stress in the spectrum. This allows the spectrum cycles to be built as fractions of the maximum stress which can be represented by the SMF. This leads to efficient and easy modification of the spectrum loading and maximum stress.

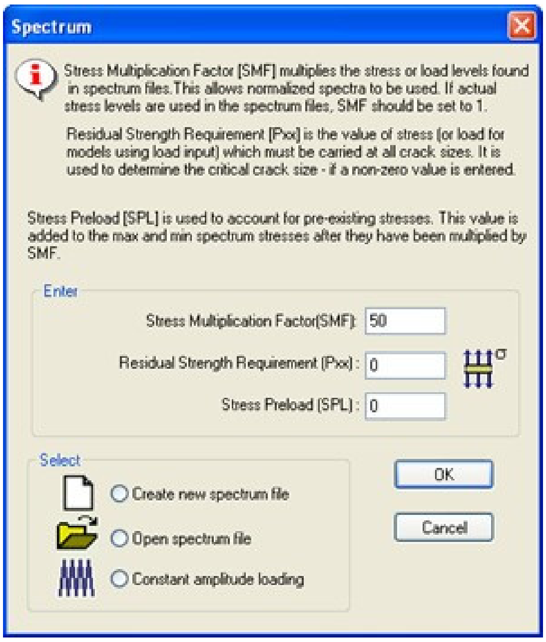

Figure 28 AFGROW spectrum input window

Spectrums created within AFGROW are created in blocks of cycles with a constant stress state, minimum and maximum stress. Constant amplitude spectrums require only the single stress ratio and number of cycles or block size. Here the block size also represents how often the crack growth is calculated and therefore also the reporting increment. Increasing the block size will reduce computation time, but will also reduce the accuracy of the analysis. For varying amplitude spectrums each stress state is entered as a fraction of the maximum stress along with the number of cycles for that stress state as seenin Figure 29. For this problem a constant amplitude spectrum with a stress ratio of zero, with a block size of 10 cycles, and a SMF of 50 "ksi" were used.

The material properties and crack growth information are jointly entered in AFGROW. For this example the Walker crack growth equation was used:

$$\mathrm{da/dN} = C\left[\Delta K(1-R)^{(\mathrm{m}-1)}\right]^{\mathrm{n}} \tag{3}$$

Where C and n are the Paris equation coefficient and n exponent respectively, m is the Walker coefficient, and R is the stress ratio. The Walker equation is an extension of Paris equation that allows for crack growth rates shifting as a function of the stress ratio [20]. The material properties and crack growth information for this example can be seen in Figure 30.

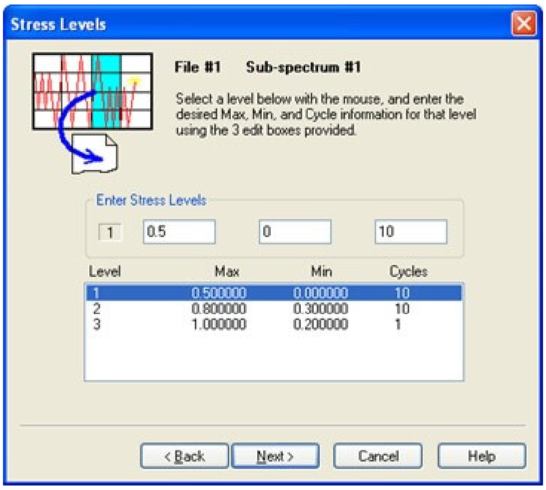

Figure 29 AFGROW created varying amplitude spectrum

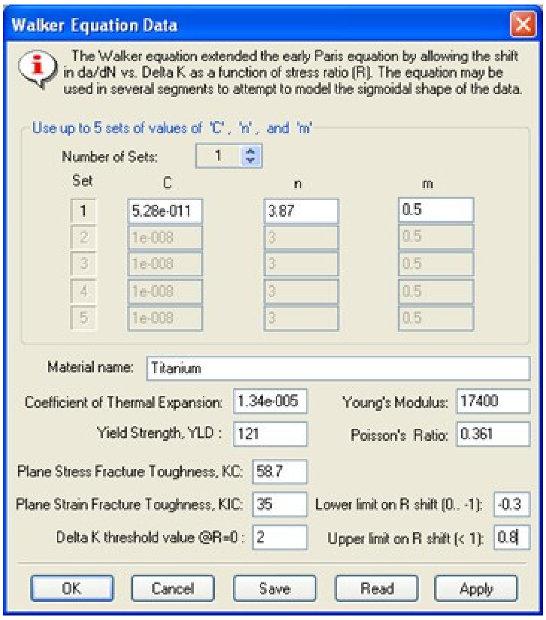

Figure 30 AFGROW titanium plate example material data

AFGROW determined this component will fail in 21470 cycles with a final crack size of 0.042839 square inches. The crack length vs. cycles can be seen in Figure 31. The number of cycles represents the number of cycles required for the center crack to propagate through the thickness of the plate in both directions. The final crack state can be seen in Figure 32, where the crack is shown as the black bar through the plate.

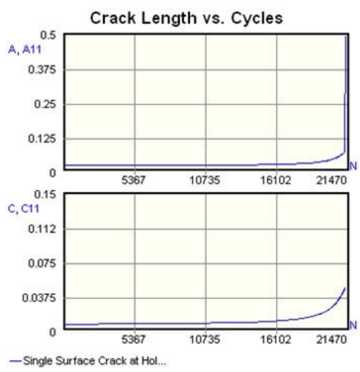

Figure 31 AFGROW titanium plate example Crack Length *vs*. Cycles Plot

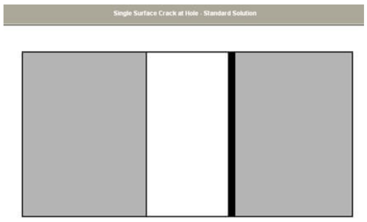

Figure 32 AFGROW titanium plate example final crack state

Zencrack is another deterministic crack growth prediction program. Developed by Zentech this program was designed to help overcome the problems that current finite element (FE) tools have with modeling and meshing cracks and crack propagation. With most current FE tools the analyst would be required to model, mesh, and run the analysis for each length of the propagating crack. Zencrack overcomes this time consuming procedure by replacing the standard brick elements around the crack with "crack-blocks." These crack-block elements surround the crack front and move with the crack during crack propagation. This allows for the growth to be fully automated in Zencrack without the need for remodeling or meshing of the crack as it grows [21]. An example of the crack-blocks after eight crack growth cycles can be seen in Figure 33.

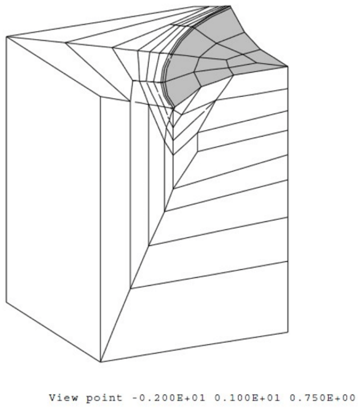

Figure 33 Meshing of one crack-block after eight crack growth increments [38]

This program is a useful tool in determining crack growth rate and cycles to failure, but, as with AFGROW this program requires the crack location and geometry be known or assumed.

However, both programs could be used to determine safe inspection intervals by calculating cycles to dysfunction, since cracks can be assumed to be the fracture critical areas for this calculation.

8 Conclusion

Safe-life is a conservative deterministic approach that retires all components based on where 1 out of a 1000 would develop a crack. Since this operating limit is determined through component testing it is up able to account for low occurrence events like service induced and inherent material defects. The damage tolerance approach accounts for these flaws by assuming that all components contain inherent flaws and the component's life is based on the time it takes for these flaws to grow to a critical size. The safe-life method in conduction with the damage tolerance approach to account for "rogue" flaws has historically provided a safe and economical approach to determining the usable life of these components. However, since both methods retire all components when a predetermined life is reach components are retired with usable life remaining. Under the retirement for cause approach components are only removed from service when there is a cause to do so. These causes include discovery of life limiting cracks and balancing the cost of future inspection with the value of the remaining usable life. This can allow the operator to extend the usable life of these critical components past the life determined by both the safe-life and damage tolerance approaches.

It is important to note that the damage tolerance and retirement for cause methods can be accomplished using either a deterministic or probabilistic method. With deterministic methods variability in loading, geometry, and material properties can only be accounted for by applying a factor of safety. This can lead to over designing the component and increasing the weight and size past acceptable limits. A probabilistic method is able to better account for this variance.

With a probabilistic approach, the deterministic solution is repeatedly calculated using random samples of variables from their determined distributions, leading to a more robust solution.

Although probabilistic methods can require a large amount of computation, this can be combated with more efficient techniques and ever advancing software. The authors believe the most cost effective way to get the most usable life out of critical components is to use the retirement for cause approach with the probabilistic method.

Acknowledgements

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