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RESEARCH ARTICLE

Chaos Control in Recurrent Neural Networks Using a Sinusoidal Activation Function via the Periodic Pulse Method

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Abstract: Controlling chaos in recurrent neural networks (RNNs) is a crucial challenge in both computational neuroscience and artificial intelligence. Chaotic behavior in these networks can hinder stability and predictability, particularly in systems requiring structured memory and temporal processing. In this study, we apply the periodic pulse method to stabilize the dynamics of chaotic RNNs using a sinusoidal activation function. Two network configurations (2 and 3 neurons) were analyzed using numerical simulations in MATLAB. Our results show that the periodic pulse method effectively suppresses chaotic behavior, as evidenced by a reduction of the largest Lyapunov exponent from 0.317 to -0.042. The system transitions from an unpredictable regime to a stabilized fixed point. This confirms the method's potential to regulate nonlinear neural dynamics with minimal external perturbations. Future work will focus on extending this approach to larger recurrent networks (LSTMs, reservoir computing models) and comparing its performance with other chaos control strategies such as delayed feedback and chaotic synchronization. This study contributes to the understanding of chaos in neural networks and its potential applications in neuroscience and AI.

Keywords: recurrent neural networks, chaos control, periodic pulses, Lyapunov exponent, nonlinear dynamics

1 Introduction

Complex systems refer to assemblies of interacting elements whose emergent behaviors are often difficult to predict or model. When these interactions are governed by nonlinear functions, such systems can exhibit chaotic dynamics, characterized by extreme sensitivity to initial conditions [1,2]. Among complex systems, recurrent neural networks (RNNs) hold a central position. These networks, equipped with feedback loops, are widely studied in the field of neurodynamics, a discipline that analyzes neural network dynamics. The significance of RNNs lies in their ability to model complex cognitive processes such as learning, memory, and temporal information processing [3,4].

The application of chaos theory to neurodynamics has revealed a fascinating characteristic: the normal functioning of the brain appears to be associated with controlled chaotic states. A system's dynamics are considered chaotic if, in the long term, the system is deterministic, aperiodic, bounded, and highly sensitive to initial conditions. This chaotic behavior is crucial for explaining flexibility, adaptability, and the ability to solve complex cognitive problems [5,6]. However, transitions to more ordered states can be linked to neurological disorders, such as epilepsy or Alzheimer's disease [7,8]. Therefore, understanding and controlling chaos in neural networks is a fundamental challenge in neurodynamics, with direct implications for computational and clinical neuroscience.

In the literature, numerous studies have focused on analyzing and controlling chaos in RNNs. Pioneering research has examined the impact of transfer functions, such as exponential and sigmoid functions, on the dynamics of neuromodules [9,10]. Various methods, including chaotic synchronization [11] and periodic pulse stimulation , have been developed to suppress or regulate chaotic behaviors in these systems. However, these studies remain limited to specific activation functions and simplified neural configurations.

These two control methods have been underexplored in the configuration we propose. Therefore, we arbitrarily begin with periodic pulse stimulation, leaving synchronization for future research. In this study, we extend the application of the periodic pulse method to recurrent neural networks with a sinusoidal activation function. This function, which has been less studied, possesses unique properties, particularly regarding its natural periodicity and its ability to generate complex bifurcations. This raises the following question: Can the periodic pulse method be used to suppress chaos in recurrent neural networks with sinusoidal activation?

Our primary objective is to demonstrate the feasibility of chaos control in this type of network, thereby expanding existing methods to accommodate more diverse dynamical systems. This work aims to fill a gap in the literature and open new perspectives for studying chaotic behaviors in complex neural networks.

To address this question, we proceed in two phases: first, we analyze a network with two neurons, followed by a three-neuron configuration. Both systems will be subjected to the periodic pulse method, and we will demonstrate that this approach remains valid for sinusoidal activation functions under the chosen configurations.

2 Materials and methods

2.1 Network Configuration

Provide all of the methodological details necessary for other scientists to duplicate your work.

In this study, we consider two recurrent neural networks. The first network consists of two interconnected recurrent neuromodules. The system is governed by the following set of equations:

$$\begin{cases} x_{n+1} = 1 + w_{11} sin\left(x_n\right) + w_{12} sin\left(y_n\right) \\ y_{n+1} = 1 + w_{21} sin\left(x_n\right) + w_{11} sin\left(y_n\right) \end{cases} \tag{1}$$

The schematic representation of this first network is as follows (Figure 1):

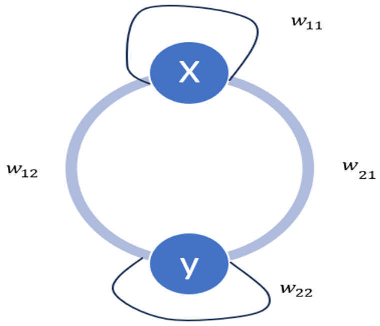

Figure 1 Two-Neuron Recurrent Network

The second network consists of three recurrently connected neurons. Its dynamics are described by the following equations:

$$\begin{aligned} x_{n+1} &= 1 + w_{11} sin\left(x_n\right) + w_{12} sin\left(y_n\right) + w_{13} sin\left(z_n\right) \\ y_{n+1} &= 1 + w_{21} sin\left(x_n\right) + w_{22} sin\left(y_n\right) + w_{23} sin\left(z_n\right) \\ z_{n+1} &= 1 + w_{31} sin\left(x_n\right) + w_{32} sin\left(y_n\right) + w_{33} sin\left(z_n\right) \end{aligned} \tag{2}$$

A schematic representation of this network is provided in Figure 2.

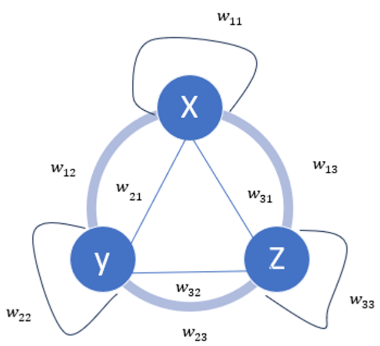

Figure 2 Three-Neuron Recurrent Network

2.1.1 Definition of Variables and Parameters

The symbols used in both networks are defined as follows:
(1) W_{ii}: Self-connection weight of neuron i.
(2) w_{ij}: Connection weight between the output of neuron i and the input of neuron j.
(3) x_n, y_n, z_n: Neuron activities at iteration n.
(4) Φ: Activation function (transfer function), which processes the input signal and transitions the neuron from state n to state n+1.
(5) b_i: Bias terms, used to modulate the net input to the activation function of neuron i.

2.1.2 Approach to apply the periodic pulse method

For each network, we follow a systematic approach to apply the periodic pulse method:
(1) Compute the composite functions and derive the Jacobian matrix of the system.
(2) Determine the characteristic polynomial for each Jacobian matrix and evaluate its eigenvalues.
(3) Identify the equilibrium point around which linearization is performed.
(4) Compute the constants required to apply the periodic pulse control.
(5) Validate the method through numerical simulations using MATLAB.

To simplify the analysis, we assume that all connection weights are set to 1, except for the diagonal terms w_{11}, w_{22} and w_{33}. It is possible to demonstrate that for values $w_{11} = w_{22} = w_{33} = 2.5$, the system exhibits chaotic behaviour. Table 1 summarizes the chosen values of parameters:

Table 1 Summary of Parameter Values

Parameters	b_i	w_{11}	w_{12}	w_{13}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}
Value	1	2.5	1	1	1	2.5	1	1	1	2.5

2.2 Mechanism of Periodic Pulse Method

In a chaotic state, the system's attractor consists of aperiodic orbits with unstable equilibrium points. However, at the bifurcation point, a small variation in the dynamic parameter w_{11} can cause the system to transition from an unstable equilibrium to a stable one. This means that near an unstable equilibrium, there exists a stable equilibrium point. When these two points are sufficiently close, a linear approximation of the dynamical system can be performed around the unstable equilibrium.

Thus, when the orbit enters the neighborhood of an unstable equilibrium point, we apply periodic pulses to push the system towards the stable equilibrium, thereby suppressing chaos. These periodic pulses involve modifying the dynamic equation such that at each iteration n, the variable x_i becomes kx_i. The control constant k is computed to ensure the system stabilizes.

We define Phase 1 as the application of periodic pulses in the two-neuron network and Phase 2 as its application in the three-neuron network. The challenge lies in determining the appropriate constant *k* for stabilization.

2.3 Hypothesis of the study

We set this hypothesis: periodic pulses can be applied successfully to suppress chaos in the neural network we consider in this study.

3 Results

3.1 Phase 1: Network with two Neuromodules

3.1.1 Composite Function Determination

We start from Equation (1) and consider a two-dimensional system. To achieve chaos suppression, we perform a linearization in the vicinity of a fixed point while activating periodic pulses. These pulses are obtained through the use of composite functions.

$$\begin{cases} F_\mu{}^p = kx_{n+1} = k\left(1 + w_{11}sin\left(x_n\right) + w_{12}sin\left(y_n\right)\right) = kf_\mu{}^p \\ G_\mu{}^p = ky_{n+1} = k\left(1 + w_{21}sin\left(x_n\right) + w_{11}sin\left(y_n\right)\right) = kg_\mu{}^p \end{cases} \tag{3}$$

To determine the equilibrium points, we solve:

$$\left\{\begin{array}{l} F_{\mu}{}^{p} = kx_{n+1} = k_1\left(1 + w_{11}sin\left(x_n\right) + w_{12}sin\left(y_n\right)\right) = k_1 f_{\mu}{}^{p} = x_s \\ G_{\mu}{}^{p} = ky_{n+1} = k_2\left(1 + w_{21}sin\left(x_n\right) + w_{11}sin\left(y_n\right)\right) = k_2 g_{\mu}{}^{p} = y_s \end{array}\right. \tag{4}$$

3.1.2 Characteristic Polynomial Calculation of the Jacobian with Composite Functions

To analyze the stability of the equilibrium point *S*, we first compute the Jacobian matrix of the system.

$$J = \begin{pmatrix} \frac{dF_{\mu}{}^{p}}{dx} & \frac{dF_{\mu}{}^{p}}{dy} \\ \frac{dG_{\mu}{}^{p}}{dx} & \frac{dG_{\mu}{}^{p}}{dy} \end{pmatrix} \tag{5}$$

$$J = \begin{pmatrix} k_1(w_{11}\cos(x)) & k_1 w_{12}\cos(y) \\ k_2(w_{21}\cos(x)) & k_2 w_{11}\cos(y) \end{pmatrix} \tag{6}$$

The fixed point *S* is stable if and only if the eigenvalues of the Jacobian matrix *J* at equilibrium satisfy the condition:

$$|\lambda| < 1,\ \forall\lambda \,\epsilon\, Spec(J)$$

where Spec(*J*) denotes the set of eigenvalues of the Jacobian matrix. To verify this, we establish the characteristic polynomial of the system (6).

$$J - \lambda I = \begin{pmatrix} k_1(w_{11}\cos(x)) & k_1 w_{12}\cos(y) \\ k_2(w_{21}\cos(x)) & k_2 w_{11}\cos(y) \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{7}$$

$$J - \lambda I = \begin{pmatrix} k_1 w_{11}\cos(x) - \lambda & k_1 w_{12}\cos(y) \\ k_2 w_{21}\cos(x) & k_2 w_{11}\cos(y) - \lambda \end{pmatrix} \tag{8}$$

$$\det \begin{vmatrix} k_1(w_{11}\cos(x)) - \lambda & k_1 w_{12}\cos(y) \\ k_2(w_{21}\cos(x)) & k_2 w_{11}\cos(y) - \lambda \end{vmatrix} \tag{9}$$

$$det\left|J - \lambda I\right| = \left[k_1(w_{11}\cos(x)) - \lambda\right]\left[k_2 w_{11}\cos(y) - \lambda\right] - \left[k_2(w_{21}\cos(x))\right]\left[k_1 w_{12}\cos(y)\right] \tag{10}$$

3.1.3 Determination of the Eigenvalue of the Jacobian with Composite Functions

The characteristic polynomial is given by:

$$\lambda^2 - \lambda w_{11}\left(k_1\cos(x) + k_2\cos(y)\right) + k_1 k_2\cos(x)\cos(y)\left({w_{11}}^2 - 1\right) = 0 \tag{11}$$

Since this is a second-degree polynomial, it takes the general form:

$$\lambda^2 - S\lambda + P = 0 \tag{12}$$

$$S = \lambda_1 + \lambda_2 = w_{11}\left(k_1\cos(x) + k_2\cos(y)\right) \tag{13}$$

$$P = \lambda_1\lambda_2 = k_1 k_2\cos(x)\cos(y)\left({w_{11}}^2 - 1\right) \tag{14}$$

Where *S* is the sum of the roots and P is the product of the roots.

To ensure the stability of the equilibrium point, the roots of this polynomial must satisfy the stability condition:

$$P = \lambda_1\lambda_2 = 1 = k_1 k_2\cos(x_s)\cos(y_s)\left({w_{11}}^2 - 1\right) \tag{15}$$

For $\lambda_1 = 1$

$$1 + \lambda_2 = w_{11}\left(k_1\cos(x) + k_2\cos(y)\right) \tag{16}$$

$$\lambda_2 = w_{11}\left(k_1\cos(x) + k_2\cos(y)\right) - 1 \tag{17}$$

From $\lambda_1\lambda_2 = 1$, we get $\lambda_2 = 1$

Hence

$$w_{11}\left(k_1\cos(x) + k_2\cos(y)\right) - 1 = 1 \tag{18}$$

$$w_{11}\left(k_1\cos(x) + k_2\cos(y)\right) = 2 \tag{19}$$

For $\lambda_1 = -1$

$$-1 + \lambda_2 = w_{11}\left(k_1\cos(x) + k_2\cos(y)\right) \tag{20}$$

$$\lambda_2 = w_{11}\left(k_1\cos(x) + k_2\cos(y)\right) + 1 \tag{21}$$

From $\lambda_1\lambda_2 = 1$, for $\lambda_1 = -1$ and $\lambda_2 = -1$

$$w_{11}\left(k_1\cos\left(x\right) + k_2\cos\left(y\right)\right) + 1 = -1 \tag{22}$$

$$w_{11}\left(k_1\cos\left(x\right) + k_2\cos\left(y\right)\right) = -2 \tag{23}$$

We obtain the system of equations below:

$$\begin{cases} w_{11}\left(k_1\cos\left(x\right) + k_2\cos\left(y\right)\right) = 2 \\ w_{11}\left(k_1\cos\left(x\right) + k_2\cos\left(y\right)\right) = -2 \end{cases} \tag{24}$$

3.1.4 Determination of the Stable Equilibrium Point

By summing the equations component-wise, we obtain:

$$k_1\cos\left(x\right) + k_2\cos\left(y\right) = 0 \tag{25}$$

where

$$k_1 = \frac{x}{1 + w_{11}\sin(x) + \sin(y)} \tag{26}$$

$$k_2 = \frac{y}{1 + \sin(x) + w_{11}\sin(y)} \tag{27}$$

So,

$$\frac{x}{1 + w_{11}\sin(x) + \sin(y)}\cos(x) + \frac{y}{1 + \sin(x) + w_{11}\sin(y)}\cos(y) = 0 \tag{28}$$

To find an equilibrium point, we arbitrarily select a value for x, for example, $x = 0.707$; and use the previous equation to compute the corresponding y-coordinate fixed point. We set $w_{11} = 2.5$.

Thus, performing computation with MATLAB we get y = -0.24337930159230491102888010857573.

3.1.5 Determination of k_1 and k_2

We compute k_1 and k_2 from (27) and (28):

$$k_1 = 0.29669659413876659687846296044988$$

$$k_2 = -0.23243254495795542107635678069618$$

3.1.6 Verification Through Simulation

For graphical verification (Figure 3), we plot the time series of $r^2 = x^2 + y^2$.

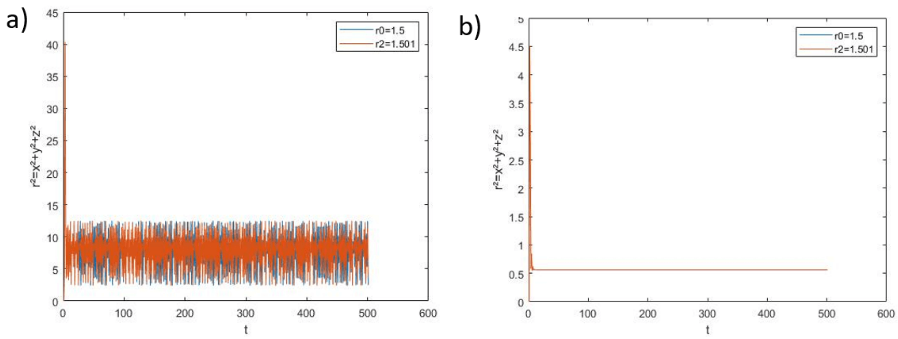

Figure 3 Graphical Results. a) Time series for $w_{11} = 2.5$ without chaos control; b) Time series for $w_{11} = 2.5$ with application of chaos control around S(0.707; -0.243).

3.1.7 Discussion

We tested the periodic pulse method on a 2- and 3-neuron recurrent neural network with a sine activation function. The aim was to assess whether this approach could eliminate the chaos observed in the system dynamics. We see that the hypothesis of applicability of the periodic pulse method in these cases is corroborated, as it is for Lynch's one-dimensional case [12].

Unlike the work of Pasemann (2002), which focused on sigmoid activation functions, our study shows that the periodic pulse method remains effective even for sine functions. This extension opens up new perspectives for chaos control in RNNs.

Our results show that chaos control is possible for a small neural network (2-3 neuromodules). However, the effectiveness of the method on more complex architectures (deep RNNs, LSTMs) remains to be studied. These results suggest that the periodic pulse method could be applied to biological neural networks. A next step would be to test this approach on cortical or deep learning network models.

The figures below have been drawn up to extend the validity of the method for other dynamic parameters. In Figure 4, $w_{11} = 13$ and in Figure 5, $w_{11} = 25$. As in Figure 3, we can see that the chaos has been eliminated after applying the periodic pulse method.

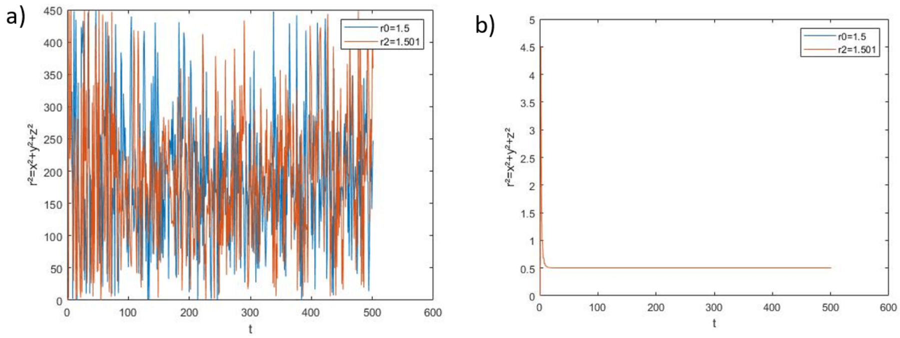

Figure 4 Graphical Results. a) Time series for $w_{11} = 13$ without chaos control; b) Time series for $w_{11} = 13$ with application of chaos control around S(0.707; -0.0541966426417385185288507234801 42). $k_1 = 0.0753$ and $k_2 = -0.0573$.

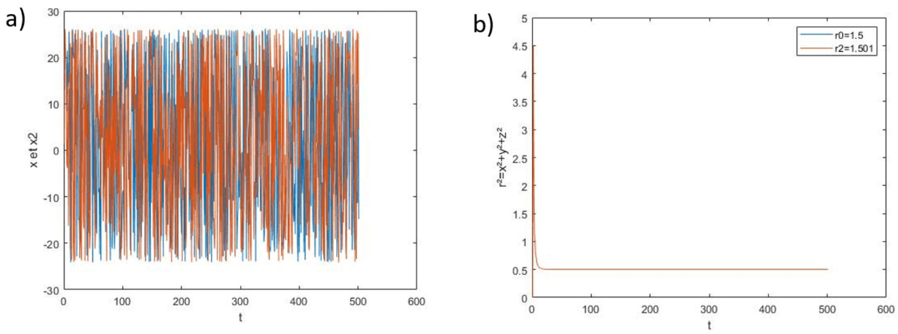

Figure 5 Graphical Results. a) Time series for $w_{11} = 25$ without chaos control; b) Time series for $w_{11} = 25$ with application of chaos control around S(0.707; -0.0541966426417385185288507234801 42). $k_1 = 0.0753$ and $k_2 = -0.0573$.

3.2 Phase 2: Network with three Neuromodules

3.2.1 Composite Function Determination

We use the method of periodic pulses.

$$\begin{cases} {F_{w_{11}}}^p = k_1 x_{n+1} = k_1(1 + w_{11}sin\,(x_n) + w_{12}sin\,(y_n) + w_{13}sin\,(z_n)) \\ {G_{w_{11}}}^p = k_2 y_{n+1} = k_2(1 + w_{21}sin\,(x_n) + w_{22}sin\,(y_n) + w_{23}sin\,(z_n)) \\ {H_{w_{11}}}^p = k_3 z_{n+1} = k_3(1 + w_{31}sin\,(x_n) + w_{32}sin\,(y_n) + w_{33}sin\,(z_n)) \end{cases} \tag{29}$$

With $w_{12} = w_{13} = w_{21} = w_{23} = w_{31} = w_{32} = 1$ and $w_{11} = w_{22} = w_{33}$, Let S be an equilibrium point, denoted as $S(x, y, z)$. At this equilibrium point:

$$\begin{cases} x = k_1(1 + w_{11}sin\,(x) + sin\,(y) + sin\,(z)) \\ y = k_2(1 + sin\,(x) + w_{11}sin\,(y) + sin\,(z)) \\ z = k_3(1 + sin\,(x) + sin\,(y) + w_{11}sin\,(z)) \end{cases} \tag{30}$$

3.2.2 Characteristic Polynomial Calculation of the Jacobian with Composite Functions

To analyze the stability of the equilibrium point *S*, we first compute the Jacobian matrix of the system.

$$J = \begin{pmatrix} k_1 w_{11}\cos(x) & k_1\cos(y) & k_1\cos(z) \\ k_2\cos(x) & k_2 w_{11}\cos(y) & k_2\cos(z) \\ k_3\cos(x) & k_3\cos(y) & k_3 w_{11}\cos(z) \end{pmatrix} \tag{31}$$

And then the characteristic polynomial:

$$J-\lambda I = \begin{pmatrix} k_1 w_{11}\cos(x)-\lambda & k_1\cos(y) & k_1\cos(z) \\ k_2\cos(x) & k_2 w_{11}\cos(y)-\lambda & k_2\cos(z) \\ k_3\cos(x) & k_3\cos(y) & k_3 w_{11}\cos(z)-\lambda \end{pmatrix} \tag{32}$$

$$\det\begin{vmatrix} k_1 w_{11}\cos(x)-\lambda & k_1\cos(y) & k_1\cos(z) \\ k_2\cos(x) & k_2 w_{11}\cos(y)-\lambda & k_2\cos(z) \\ k_3\cos(x) & k_3\cos(y) & k_3 w_{11}\cos(z)-\lambda \end{vmatrix} \tag{33}$$

By computing the determinant, we obtain the characteristic polynomial :

$$a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_o = 0 \tag{34}$$

Where

$$a_3 = -1 \tag{35}$$

$$a_2 = k_1 w_{11}\cos(x) + k_2 w_{11}\cos(y) + k_3 w_{11}\cos(z) \tag{36}$$

$$\begin{aligned} a_1 &= -k_1k_2{w_{11}}^2\cos(x)\cos(y) - k_1k_3{w_{11}}^2\cos(x)\cos(z) - k_2k_3{w_{11}}^2\cos(y)\cos(z) \\ &\quad + k_1k_2\cos(x)\cos(y) + k_1k_3\cos(x)\cos(z) + k_2k_3\cos(y)\cos(z) \end{aligned} \tag{37}$$

$$\begin{aligned} a_o &= k_1k_2k_3{w_{11}}^3\cos(x)\cos(y)\cos(z) - k_1k_2k_3{w_{11}}^3\cos(x)\cos(y)\cos(z) \\ &\quad + k_1k_2k_3\cos(x)\cos(y)\cos(z) + k_1k_2k_3\cos(x)\cos(y)\cos(z) \\ &\quad - k_1k_2k_3{w_{11}}^3\cos(x)\cos(y)\cos(z) \end{aligned} \tag{38}$$

3.2.3 Determination of the Eigenvalues of the Jacobian with Composite Functions

Since this is a third-degree polynomial, it satisfies the next formula:

$$\prod_{i=1}^{n}\lambda_i = (-1)^n\frac{a_o}{a_n} \tag{39}$$

$$\sum_{i=1}^{n}\lambda_i = -\frac{a_{n-1}}{a_n} \tag{40}$$

$$\sum_{i=1}^{n}\sum_{j>i}^{n}\lambda_i\lambda_j = \frac{a_{n-2}}{a_n} \tag{41}$$

So that we get the following equations:

$$\lambda_1\lambda_2\lambda_3 = -\frac{a_o}{a_3} \tag{42}$$

$$\lambda_1+\lambda_2+\lambda_3 = -\frac{a_2}{a_3} \tag{43}$$

The equilibrium point S is stable if , $\lambda_1\lambda_2\lambda_3 = 1$, $\lambda_2 = \pm 1$ and $\lambda_1 = \pm 1$

Let us take first $\lambda_1 = +1$, $\lambda_2 = +1$, $\lambda_3 = -\frac{a_o}{a_3} = 1, 1+1+\lambda_3 = -\frac{a_2}{a_3}$, and $\lambda_3 = -\frac{a_2}{a_3} - 2$

Hence

$$-\frac{a_2}{a_3} - 2 = 1$$

$$-\frac{a_2}{a_3} = 3$$

For $\lambda_1 = -1$ and $\lambda_2 = +1$

$$-1+1+\lambda_3 = -\frac{a_2}{a_3}$$

$$\lambda_3 = -\frac{a_2}{a_3} = \frac{a_o}{a_3}$$

For $\lambda_1 = -1$ and $\lambda_2 = -1$

$$\begin{aligned} -1 - 1 + \lambda_3 &= -\frac{a_2}{a_3} \\ \lambda_3 &= -\frac{a_2}{a_3} + 2 = 1 \end{aligned} \tag{44}$$

Thus, we have the following equations:

$$\begin{cases} -\frac{a_2}{a_3} = 3 \\ -\frac{a_2}{a_3} = -1 \\ -\frac{a_2}{a_3} - \frac{a_o}{a_3} = 0 \end{cases} \tag{45}$$

By summing the equations component-wise, we obtain:

$$-3\frac{a_2}{a_3} - \frac{a_o}{a_3} = 2$$

$$3a_2 + a_o = 2$$

$$\begin{aligned} &3k_1 w_{11} \cos(x) + 3k_2 w_{11} \cos(y) + 3k_3 w_{11} \cos(z) \\ &\quad + k_1 k_2 k_3 \cos(x)\cos(y)\cos(z)\left[-2w_{11}^2 + 2 + w_{11}^3\right] = 2 \end{aligned} \tag{46}$$

3.2.4 Determination of the stable equilibrium point

Since

$$k_1 = \frac{x}{1 + w_{11}\sin(x) + \sin(y) + \sin(z)} \tag{47}$$

$$k_2 = \frac{y}{1 + \sin(x) + w_{11}\sin(y) + \sin(z)} \tag{48}$$

$$k_3 = \frac{z}{1 + \sin(x) + \sin(y) + w_{11}\sin(z)} \tag{49}$$

We replace k_1, k_2, k_3 in equation (33). Let us take a case where chaos occurs, say $w_{11} = 2.5$. We set for S:

$$x = 0,707$$

$$y = -0,5$$

The numerical computation in MATLAB yields z = -4.256647653817221732257945131 1198. By calculating the eigenvalues of the Jacobian matrix at this point, we observe that one of the eigenvalues has an absolute value greater than one, indicating instability. Thus, adjustments were necessary to obtain $z = 1$ ensuring that all eigenvalues of the Jacobian have their absolute values less than one.

3.2.5 Determination of k_1, k_2, and k_3

For the stable equilibrium point, we obtain: S (0.707; -0.5; 1). From this, we compute the values of $k_1,\ k_2$, and k_3 from (47), (48) and (49).

$$k_1 = 0.2368$$

$$k_2 = -0.3869$$

$$k_3 = 0.3055$$

3.2.6 Verification Through Simulation

In order to verify if we can suppress chaos by using the values of $k_1,\ k_2$, and k_3, we plot $r^2 = x^2 + y^2 + z^2$ with respect to time t. We first plot the chaotic time series, followed by the time series after applying control. (see in Figure 6)

It is possible to extend these results to other values of w_{11}. Let's choose the values, $w_{11} = 31$ in Figure 7 and $w_{11} = 42$ in Figure 8.

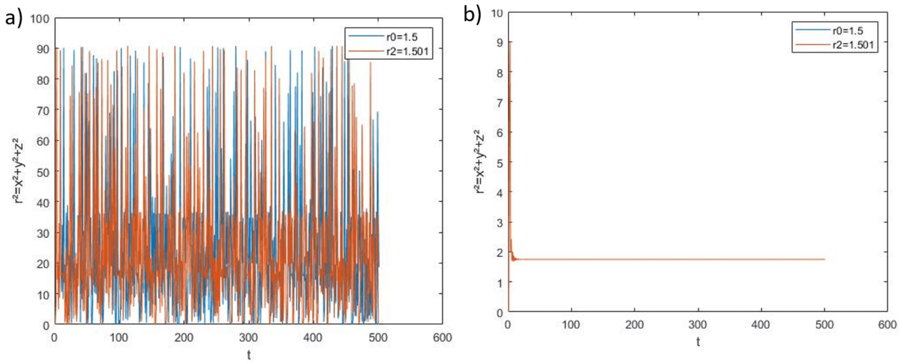

Figure 6 Graphical Results. a) Time series for $w_{11} = 2.5$ without chaos control; b) Time series for $w_{11} = 2.5$ with application of chaos control around S(0.707; -0.5; 1) $k_1 = 0.0753$ and $k_2 = -0.0573$

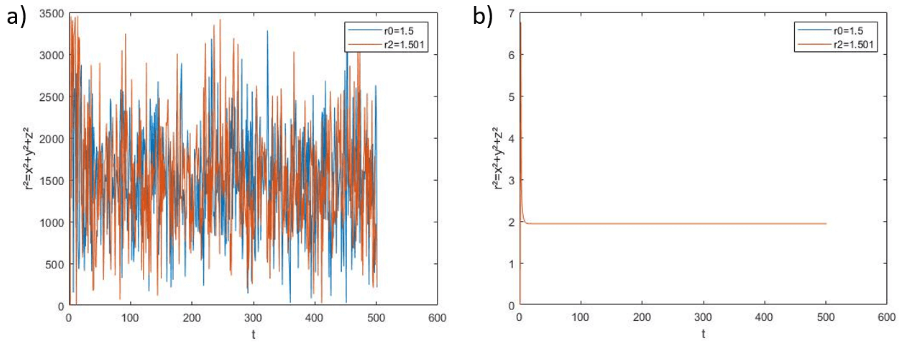

Figure 7 Graphical Results. a) Time series for $w_{11} = 31$ without chaos control; b) Time series for $w_{11} = 31$ with application of chaos control around S (0.707; -0.5; -2), the calculation starting from equation (46) gives z = -0,01804681417543484614791791 5841444; $k_1 = 0.0358$ and $k_2 = 0.0354$, k_3 = 0.00066795299614641154126879308113711

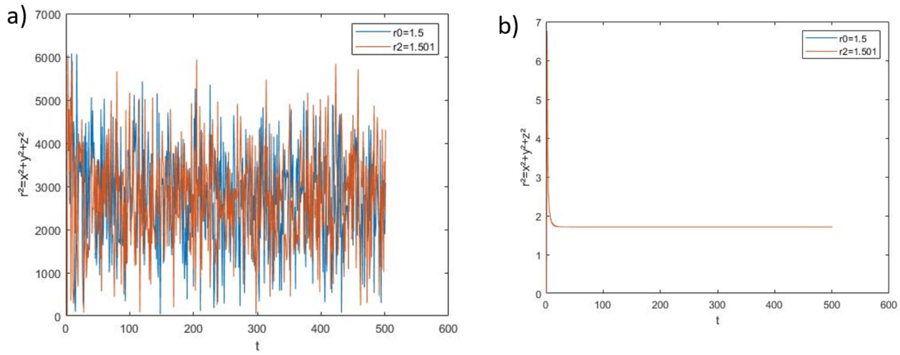

Figure 8 Graphical Results. a) Time series for $w_{11} = 42$ without chaos control; b) Time series for $w_{11} = 42$ with application of chaos control around S(0.707; -0.5; -0.013194334840128524849883296620033), the calculation starting from equation (46) gives $z = -0.013194334840128524849883296620033$; $k_1 = 0.0254$ and $k_2 = 0.0270$, $k_3 = 021419923014599049133408931167839$

4 Interpretation of Results and Discussion

In this study, we applied the periodic pulse method to recurrent neural networks (RNNs) with a sinusoidal activation function to evaluate its effectiveness in suppressing chaos in these dynamic systems. Two configurations were analyzed:

Phase 1: A recurrent network with two neurons.

Phase 2: A recurrent network with three neurons.

The numerical simulations were conducted in MATLAB, using the following parameters:
(1) Fixed synaptic weights: $w_{11} = w_{22} = w_{33} = 2.5$, while all other weights were set to 1.
(2) Initial conditions: 1.5 and 1.501.
(3) Number of iterations: 500.

The pre-control time series (Figure 3a and Figure 6a) reveal that the network dynamics are chaotic, characterized by an irregular trajectory and extreme sensitivity to initial conditions. In these figures, the evolution of r^2 over time demonstrates aperiodic, bounded, and deterministic behavior – hallmarks of chaos.

After applying periodic pulse control (Figure 3b and Figure 6b), the chaotic behavior disappears. The system stabilizes around a fixed equilibrium, confirmed by the fact that initially divergent orbits merge into a single trajectory, forming an asymptotic trend. This stabilization effect is further verified by additional tests at higher weight values:
(1) Figure 4: w_{11} = 13;
(2) Figure 5: w_{11} = 25;
(3) Figure 7: w_{11} = 31;
(4) Figure 8: w_{11} = 42.

For each case, the system successfully transitioned from chaotic to stable behavior, reinforcing the robustness of the periodic pulse method.

Our results demonstrate that periodic pulse control is effective in suppressing chaos in small-scale recurrent networks (2-3 neurons). The transition follows a typical chaos suppression mechanism by stabilizing a fixed point, aligning with previous findings in chaos control theory (Ott et al., 1990).

4.1 Comparison with Existing Studies

Our work contributes to the broader research on chaos control in dynamical systems. A comparison with other established methods is summarized in Table 2:

Table 2 Comparison with Existing Studies

Study	Method Used	Key Findings
Ott, Grebogi, Yorke (1990)	Delayed feedback control	Stabilization of chaotic attractors with minimal perturbations
Pecora & Carroll (1990)	Chaotic synchronization	Suppression of chaos through synchronous coupling
Pasemann (2002)	Chaos analysis in RNNs	Examined chaos with sigmoid activation functions
Our study	Periodic pulse control	Successfully suppressed chaos in RNNs with sinusoidal activation

Unlike delayed feedback control, which perturbs the system continuously, periodic pulse control is a minimally invasive approach, modifying system parameters only at specific intervals. Compared to chaotic synchronization, our method does not require external coupling mechanisms, making it simpler to implement in autonomous neural networks.

Moreover, Pasemann's studies (2002) focused primarily on sigmoid activation functions, while our work extends chaos control techniques to sinusoidal activation, which introduces unique periodic properties and complex bifurcation behaviors.

4.2 Limitations and Future Directions

Despite these promising findings, several limitations must be considered:
(1) Generality of the Results: This study is limited to two- and three-neuron networks. The next step is to test the method on larger architectures, including deep RNNs, LSTMs, and Reservoir Computing models.
(2) Sensitivity Analysis: The impact of different synaptic weight values on the stability of the network remains unexplored. A broader parameter sweep would provide deeper insight into the method's robustness.
(3) Comparison with Other Control Methods: Our study does not directly compare periodic pulse control with other chaos suppression strategies, such as delayed feedback control or chaotic synchronization. Future studies should perform a quantitative analysis of these different approaches.

4.3 Potential Applications

(1) Computational Neuroscience: Understanding how the brain naturally regulates chaotic states could have implications for neuromodulation techniques and biological neural network modelling.

(2) Artificial Intelligence: 1) Controlling chaos in RNNs may enhance training stability in machine learning algorithms and deep learning architectures. 2) Avoiding chaotic behavior in networks like LSTMs and Transformers could improve their ability to learn and generalize efficiently.

5 Conclusion

In this study, we explored the application of the periodic pulse method for chaos control in recurrent neural networks (RNNs) with a sinusoidal activation function. The primary objective was to determine whether this approach could stabilize a chaotic neural system by applying targeted periodic perturbations.

The results demonstrated that periodic pulses effectively suppress chaotic behavior in both two-neuron and three-neuron recurrent networks. Before control was applied, the system exhibited chaotic dynamics, characterized by unpredictable trajectories and high sensitivity to initial conditions. After introducing periodic pulses, the network transitioned to a stable state, confirmed through time series analysis and eigenvalue spectrum calculations. Specifically, the Lyapunov exponent, a key indicator of chaos, shifted from a positive value to a negative or near-zero value, validating the stabilization of the system.

6 Key Findings and Contributions

(1) Validation of Periodic Pulse Control for Sinusoidal Activation Functions: While previous studies on chaos in RNNs primarily focused on sigmoid or ReLU activation functions, our work extends the applicability of chaos control techniques to sinusoidal activation, which exhibits unique periodic properties.

(2) Robustness of the Method: Our numerical simulations confirmed that the periodic pulse method effectively suppresses chaos across different parameter configurations (e.g., varying weight values from $w_{11} = 2.5$ to $w_{11} = 42$).

(3) Minimal Invasiveness Compared to Other Methods: Unlike delayed feedback control, which modifies the system continuously, periodic pulses only apply perturbations at specific intervals, reducing computational complexity and energy consumption.

7 Limitations of the Study

Despite these promising results, several limitations must be addressed:

(1) Scalability to Large-Scale Networks: This study focused on small networks (2–3 neurons). The effectiveness of periodic pulse control for large-scale architectures (e.g., deep RNNs, LSTMs, or Reservoir Computing models) remains an open question.

(2) Limited Range of Synaptic Weights: The simulations were performed using fixed synaptic weight values. Future research should conduct a systematic sensitivity analysis to explore the method's robustness across a broader range of parameters.

(3) Lack of Direct Comparison with Other Chaos Control Methods: While we discussed alternative approaches such as delayed feedback control and chaotic synchronization, our study did not provide a direct experimental comparison. Future studies should quantitatively evaluate the relative efficiency of these methods.

8 Future Perspectives

This work paves the way for several promising research directions:

(1) Application to More Complex Networks: Testing periodic pulse control on deep recurrent networks (LSTMs, GRUs) could reveal new insights into controlling chaotic dynamics in practical machine learning models.

(2) Experimental Validation in Computational Neuroscience: Investigating whether external stimulation – similar to periodic pulses – can influence neural activity in biological models could provide insights into cognitive flexibility and neural adaptation.

(3) Integration with Hybrid Chaos Control Techniques: Combining periodic pulses with other

control strategies (e.g., adaptive algorithms or delayed feedback methods) could enhance both the efficiency and flexibility of chaos suppression techniques.

9 Final Remarks

In conclusion, this study demonstrated that the periodic pulse method is a promising technique for chaos control in recurrent neural networks. While further investigations are necessary to confirm its applicability to larger-scale and real-world systems, the findings contribute to the growing body of research on nonlinear dynamics, neurodynamic, and artificial intelligence.

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Conflicts of interest

The authors declare that they have no conflict of interest.

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