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RESEARCH ARTICLE

Material Selection for Gear Manufacture in Terms of the Probabilistic Multi-objective Optimization

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Abstract: Probabilistic multi-objective optimization based material selection is conducted for gear manufacturing. This method incorporates the new concepts of preferable probability and total preferable probability of an alternative, which are determined by comprehensively considering all possible property responses of the alternative. Each property response of a material contributes a partial preferable probability to the alternative in a linearly correlative manner, either positively or negatively, depending on whether it is a beneficial or unbeneficial type in the evaluation. The total preferable probability of an alternative is obtained by multiplying all partial preferable probabilities. The optimal choice is the alternative with the maximum total preferable probability. In gear manufacturing material selection, five criteria are considered: core hardness, surface hardness, surface fatigue limit, bending fatigue limit, and ultimate tensile strength. Core hardness is regarded as an unbeneficial response, while the other four are beneficial. Through quantitative assessment, carburized steel is ultimately chosen as the optimal material.

Keywords: gear manufacture, material selection, quantitative assessment, preferable probability, multi-object optimization

1 Introduction

A systematic and quantitative method for material selection is crucial for effective material design and application in practical engineering, especially when dealing with a material database containing a vast amount of data [1].

Since the pioneering work of Ashby [2, 3], numerous methods have been developed to analyze material property data to achieve rational and systematic results [1–5]. However, material selection is inherently challenging [1–3], as it involves multiple material properties such as strength, ductility, fatigue resistance, and corrosion resistance, some of which may even conflict with each other. Therefore, decisions on material selection and substitution require a comprehensive consideration of all relevant material properties to achieve a balanced "trade-off" solution. This indicates that material selection is essentially a multi - objective optimization problem.

Recently, probabilistic multi – objective optimization (PMOO), developed from a systems theory perspective [6], has introduced new concepts of preferable probability and total preferable probability. Each material property contributes a partial preferable probability to the alternative in a linearly correlative manner, either positively or negatively, depending on whether it is a beneficial or unbeneficial type in the evaluation. The partial preferable probability of each property with the same physical meaning is normalized within the alternative material group. To fully consider the simultaneity of all property responses in the evaluation, the multiplication of all partial preferable probabilities produces the total preferable probability of an alternative material. This total preferable probability serves as the sole indicator reflecting the material's overall property response. Consequently, the alternative material with the highest total preferable probability is the optimal choice.

PMOO offers several advantages. It avoids the confusing problems found in other approaches [6, 7], such as the unreasonable "additive operation" of different property responses and the subjective choice of normalization factors for each property response in other multi - objective optimizations (MOO), as well as the irrational or non - quantitative statements in empirical approaches [6, 7]. In this paper, PMOO is applied to material selection for gear manufacturing.

2 Concise Introduction of PMOO

Some properties are beneficial to an optimal option, following a "the higher, the better" principle, while others are detrimental, following a "the lower, the better" principle. Most actual alternatives embody both beneficial and detrimental properties and cannot be purely one or the other. Thus, a comprehensive, impersonal analytical approach is essential. Fortunately, PMOO meets this need for multi - attribute optimization [6,7]. In the PMOO approach [6,7], the new concept of preferable probability was developed to represent the preferable degree of the property response in the option competition comparatively and quantitatively. Furthermore, quantification of preferable probability is conducted.

It assumed that the preferable probability of a property response with the characteristic of beneficial responses in the option process is correlated to the utilization of this property response positively in linear manner [6,7], *i.e.*,

$$P_{\alpha\beta} \propto Y_{\alpha\beta}, P_{\alpha\beta} = A_\beta Y_{\alpha\beta}, \alpha = 1, 2, \ldots, r, \beta = 1, 2, \ldots, s. \tag{1}$$

In Eq. (1), $Y_{\alpha\beta}$ reflects the utilization of this property response of the β-*th* property response of the α-*th* alternative; $P_{\alpha\beta}$ is the partial preferable probability of the beneficial property response $Y_{\alpha\beta}$; r is the total number of alternatives in the option group involved; *s* is the total number of property responses of each alternative in the group; A_β is the normalized factor of the β-*th* property response.

Moreover, it obtained [6,7],

$$\sum_{\alpha=1}^{r} A_\beta Y_{\alpha\beta} = \sum_{\alpha=1}^{r} P_{\alpha\beta} = 1,\ A_\beta = 1/(n\overline{Y_\beta}) \tag{2}$$

$\overline{Y}_\beta$ is the average value of the utilization of the β-*th* property response in the alternative group involved.

Analogically, partial preferable probability of the unbeneficial property response $Y_{\alpha\beta}$ of the alternative is correlated to its utilization of this property response negatively in linear manner, *i.e.*,

$$P_{\alpha\beta} \propto (Y_{\beta\max}+Y_{\beta\min}-Y_{\alpha\beta}),\ P_{\alpha\beta} = B_\beta(Y_{\beta\max}+Y_{\beta\min}-Y_{\alpha\beta}),\ \alpha = 1, 2, ..., r,\ \beta = 1, 2, ..., s. \tag{3}$$

In Eq. (3), $Y_\beta max$ and $Y_\beta min$ indicate the maximum and minimum values of the utilization of the property response Y_β in the alternative group, respectively; B_β is the normalized factor of the β-*th* property response. Correspondingly, it obtained [6,7],

$$B_\beta = 1/[r(Y_{\beta\max} + Y_{\beta\min}) - r\overline{Y_\beta}] \tag{4}$$

Subsequently, the total / comprehensive preferable probability of the α-*th* alternative to is the product of its all possible partial preferable probability $P_{\alpha\beta}$ of each property responses, *i.e.*,

$$P_\alpha = P_{\alpha 1} \cdot P_{\alpha 2} \cdots P_{\alpha s} = \prod_{\beta=1}^{s} P_{\alpha\beta} \tag{5}$$

Finally, the total preferable probability P_α of the $\alpha - th$ alternative is the decisive indicator for the option to conduct the competition comparatively, the winner / victor is with the maximum total preferable probability.

As the weighting factor w_β is considered, Eq. (5) is alternatively modified as [6,7],

$$P_\alpha = {P_{\alpha 1}}^{w_1} \cdot {P_{\alpha 2}}^{w_2} \cdots {P_{\alpha m}}^{w_s} = \prod_{\beta=1}^{s} {P_{\alpha\beta}}^{w_\beta} \tag{6}$$

Impersonally, the weighting factor w_β could be assessed by Eq. (7) [6,7],

$$w_\beta = \frac{C_\beta}{\left(\sum_{\beta=1}^{s} C_\beta\right)},\ C_\beta = \left\{ \frac{\left[\sum_{\alpha=1}^{r} \left(P_{\alpha\beta} - \frac{1}{r}\right)^2\right]}{r} \right\}^{0.5} \tag{7}$$

Eq. (7) indicates that the bigger the variation of the partial preferable probabilities of the β-*th* property response from alternative to alternative the bigger the weighting factor w_β is.

It is sure, in some cases the weighting factors are decided artificially by according to subjective preference of evaluators or experts. In addition, the probabilistic robust design of production process and product was developed [8].

3 Utilization of the PMOO in Material Selection of Gear Manufacture

Milani et al. once proposed a problem of material option for gear manufacture [9–12]. Material selection for gear manufacture is a typical optimal option problem with multiple property responses conflicting each other. In the study of Milani et al. [9–12], there were nine materials as the alternatives for the gear manufacture, *i.e.*, ductile iron, cast iron, SG iron, through hardened alloy steel, cast alloy steel, surface hardened alloy steel, nitride steel, through hardened carbon steel and carburized steel, which are coded by S_α (α = 1, 2, . . . , 9) . The property responses of those nine alternative materials was evaluated with respect to five selection criteria, *i.e.*, core hardness (C), surface hardness (S), surface fatigue limit (F), bending fatigue limit (B), and ultimate tensile strength (U). Among these five criteria, the responses of S, F, B, and U are in beneficial type, while response of C is in unbeneficial type in the preference assessment of the option.

Table 1 displays the property responses of the alternatives in the gear manufacture. The alternatives shown in Table 1 form an alternative group for the option. Table 2 gives the assessed results of the partial probabilities of the property responses of alternative materials for the gear manufacture. Table 3 represents the assessed results of the impersonal weighting factors of the property responses of alternative materials for the gear manufacture. The final evaluated results of the total preferable probabilities and ranking are given in Table 4.

Table 1 Property responses of alternative materials for the gear manufacture [9–12]

Material \ Property	C (Bhn)	S (Bhn)	F (N/mm^2)	B (N/mm^2)	U (N/mm^2)
Ductile iron (S_1)	220	220	460	360	880
Cast iron (S_2)	200	200	330	100	380
SG iron (S_3)	240	240	550	340	845
Through hardened alloy steel (S_4)	270	270	670	540	1190
Cast alloy steel (S_5)	270	270	630	435	590
Surface hardened alloy steel (S_6)	240	585	1160	680	1580
Nitride steel (S_7)	315	750	1250	760	1250
Through hardened carbon steel (S_8)	185	185	500	430	635
Carburized steel (S_9)	315	700	1500	920	2300

Table 2 Partial probability of the property responses of alternative materials for the gear manufacture

Material \ Probability	P_C	P_S	P_F	P_B	P_U
S_1	0.1277	0.0643	0.0652	0.0789	0.0912
S_2	0.1386	0.0585	0.0468	0.0219	0.0394
S_3	0.1168	0.0702	0.0780	0.0745	0.0876
S_4	0.1005	0.0789	0.0950	0.1183	0.1233
S_5	0.1005	0.0789	0.0894	0.0953	0.0611
S_6	0.1168	0.1711	0.1645	0.1490	0.1637
S_7	0.0761	0.2193	0.1773	0.1665	0.1295
S_8	0.1467	0.0541	0.0709	0.0942	0.0658
S_9	0.0761	0.2047	0.2128	0.2015	0.2383

Table 3 Weighting factors of the property responses of alternative materials for the gear manufacture

Property	C	S	F	B	U
Weighting factor, w_j	0.0943	0.2520	0.2192	0.2040	0.2304

Table 4 Assessed results of the total preferable probabilities and ranking of alternative materials for the gear manufacture

Alternative material	Total preferable probability	Ranking
S_1	0.0778	7
S_2	0.0451	9
S_3	0.0803	6
S_4	0.1012	4
S_5	0.0813	5
S_6	0.1575	3
S_7	0.1586	2
S_8	0.0739	8
S_9	0.1941	1

The last column of Table 4 shows that the comparative consequence shows clearly that alternative S_9, *i.e.*, carburized steel,exhibitsthe maximum value of total preferable probability, so the optimal option in material selection for gear manufacture is carburized steel by means of PMOO.

4 Conclusion

As discussed, PMOO offers a comprehensive method to account for all possible material property responses when optimally selecting gear manufacturing materials. Five criteria are considered, and the total preferable probability determines the final material choice. After detailed quantitative evaluation, carburized steel emerges as the optimal material due to its maximum total preferable probability.

Conflicts of interest

The authors declare that they have no conflict of interest.

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