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RESEARCH ARTICLE

Probabilistic Multi-objective Optimization of Aromatic Extraction Process

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Abstract: Aromatics extraction is a crucial step in the aromatics production process. Optimization of aromatics extraction process is of great significance for enhancing the overall efficiency of the aromatics unit system with minimizing process energy consumption. The purity and energy consumption of a product are fundamental metrics that need to be optimized simultaneously, which thus make it a multi-objective optimization problem. However, a careful analysis reveals that previous multi-objective optimization methods lack a clear perspective despite having algorithms. This article provides a procedure for maximizing product purity and minimizing process energy consumption during aromatic extraction by means of probabilistic multi-objective optimization (PMOO) together with regression so as to supply optimum parameters of aromatic extraction. PMOO is based on the viewpoint of systems theory, which adopts the method of probability theory to deal with the problem of simultaneous optimization of multiple objectives, and introduces the concept of "preferable probability", it establishes a methodology of probabilistic multi - objective optimization. The evaluated objectives of the candidate in the optimization task are preliminarily divided into two basic types, *i.e.*, the beneficial type and the unbeneficial (cost) type, and the corresponding quantitative evaluation methods of partial preferable probabilities are formulated for these two types. Taking the overall optimization of the optimal problem as a system, the simultaneous optimization of multiple attributes is analyzed as the simultaneous occurrence of multiple events in probability theory. Therefore, the total preferable probability of each alternative candidate is the product of partial preferable probabilities of all possible attributes of the alternative candidate, which optimizes the system as a whole. Finally, all alternative candidates are sorted and optimized according to their values of the total preferable probabilities. Beside, in the optimization, the functional relationship between the total preferable probability and input variables is regressed to get the optimum status and corresponding parameters with reliable limited test data. This method opens up a new way to solve multi-objective problems and has broad application prospects.

Keywords: systems theory, probability theory, aromatic extraction, multi-objective optimization, preferable probability

1 Introduction

Aromatic hydrocarbon extraction is a necessary link in aromatic hydrocarbon production process, and optimizing this process is of great significance for improving the benefit of the whole aromatic hydrocarbon plant system. Product purity and energy consumption are the basic indexes to be optimized in the process of aromatic hydrocarbon extraction, so it belongs to "multi-objective optimization" problems.

In the 1960s and 1970s, the optimization method promoted by Prof. Hua Luogeng and his group spread all over 28 provinces, municipalities and autonomous regions in China, and achieved a lot of results and good economic benefits. Subsequently, it provided consultation for the long-term planning, major project research and financing of the country and industry. The work of Prof. Hua Luogeng and his group has also attracted the attention and reference of international counterparts, which can be called a model of mathematical application [1]. While with the increase of the number of objectives many objectives (attributes) need to be optimized at the same time in a system, "multi-objective optimization" methods appeared [2]. For objective management and planning, it is derived from linear programming.

In 1961, A. B. Charnes and W. W. Cooper put forward relevant concepts and models when considering the approximate solution of infeasible linear programming problems [3]. At present, some solutions have been developed, each is with its own characteristics. For the optimization (option) problem of a system with multiple attributes, the optimization criteria may usually conflict each other, and it is always hoped to adopt the “multi-objective optimization” method to solve this kind of problem.

At present, some multi-objective optimization methods have been developed, Such as: simple additive weighting (SAW) [4], weighted aggregation and product evaluation (WASPAS) [5], preference method similar to ideal solution (TOPSIS) [6], VIKOR method (VLSEKriterijumska optimizacija i komprominisno Resenje) [7], analytic hierarchy process (AHP), Multi-objective optimization method based on ratio analysis (MORA) [8], Composite Proportion Assessment (COPRAS) [9], Proximity Index Value (PIV) [10], Preference Selection Index (PSI) [11], Preference Selection Index (PSIE) determined by entropy method [12], etc. These methods have been applied in many fields to varying degrees. However, above methods have their fundamental defects. For example, in the linear weighting method, if the objective functions $f_1(x)$, $f_2(x)$, ..., $f_p(x)$ are “added” with the weighting coefficient w_j, the option of the weight coefficient is subjective instead of objective one, and the attributes need to be “normalized” when the dimensions of the attributes are different, and the rationality of the normalized denominator is unclear. Some methods also introduce artificial factors such as virtual “ideal point”. Not only that, fundamentally speaking, in set theory, “addition” algorithm is union; in probability theory, “addition” is the “sum” of events. Obviously, it can be seen that the operation mode of “addition” fundamentally deviates from the original intention of “simultaneous optimization”. Pareto solution can only give a set of solutions. It can be seen that the method of multi-objective (attribute) optimization is not perfect. In fact, the original intention of multi-objective optimization (optimization) is to “optimize multiple objectives” simultaneously. From the perspective of probability theory, it is the “product” of probability of each objective; in set theory, it belongs to the “intersection” of various objectives.

In view of the above situation, in recent years, the multi-objective optimization problem can be regarded as an optimal problem in a system from the viewpoint of systems theory. According to the viewpoint of systems theory, “the optimization point of multiple objectives” in a system is “the optimum point of the whole system”. Then, the multi-objective optimization problem could be solved by using the methods of set theory and probability theory, and the new concept of “preferable probability” is introduced [12], and thus the theory and methodology of probabilistic multi-objective optimization (PMOO) is thus formulated. Furthermore, the evaluation objectives (attributes) of the candidates in the optimization task are preliminarily divided into two basic types: beneficial type of attribute and unbeneficial (cost) type of attribute, and quantitative evaluations of partial preferable probability corresponding to types of either beneficial or unbeneficial attribute are regulated [12]. Taking the whole optimization as a system and carrying out “simultaneous optimization of multiple attributes” is a problem of “simultaneous occurrence of multiple events” in probability theory. Therefore, the total preferable probability of each candidate objective is the product of partial preferable probabilities of all possible attributes of the candidate objective. Thus the overall optimization of the system can be handled. Finally, all the candidate objectives are ranked according to their total preferable probability, which is the unique and decisive basis for the candidate objective to win the competition in this optimization. Deeper analysis shows that there are obvious differences between the above methods. It indicates that “probabilistic multi-objective optimization” has both viewpoint and algorithm. According to the viewpoint of systems theory, it concludes that “the optimal point of multi-objective optimization” is “the optimum point of the system”, and then this optimum point of the system is obtained by probability theory. However, in the other previous “multi-objective optimization methods” in the past, there are “only algorithms instead of opinions”, that is, which lacks the definition of optimum point of “multi-objective optimization”.

In this paper, the probability-based multi-objective optimization method is utilized together with regression to solve the problems of parameter optimization and production scheme determination of aromatic extraction, so as to establish a more effective method. The remarkable feature of this method is to optimize multiple objectives at the same time in preferable probability, without any artificial or subjective scaling factor, which opens up a new way to solve multi-objective problems and has broad application prospects.

2 Overview of probabilistic multi-objective optimization method

The original intention of multi-objective optimization is to optimize these “multiple objectives” in a system at the same time. From the perspective of probability theory, it is the "product" of the probability of each objective and the "intersection" of each objective in set theory. So the task now is to introduce an "analog" of probability like in probability, thus the new concept of "preferable probability" was introduced to handle the matter. The evaluated objectives (attributes) of candidates in the optimization task are preliminarily divided into two basic types, *i.e.*, beneficial type of attribute and unbeneficial (cost) type of attribute, and a quantitative evaluation methods of partial preferable probability corresponding to both beneficial and unbeneficial (cost) types of attributes are established by taking the "optimization of multiple attributes at the same time" as the whole optimization of a system [12]. In probability theory, it is treated as "multiple events appear at the same time". Therefore, the total preferable probability of each candidate objective is the product of partial preferable probabilities of all possible attributes of the candidate objective to get the overall optimization of the system. Finally, the total preferable probability of each candidate is the unique and decisive index for the candidate to win the competition in this optimization. The process of probabilistic multi-objective optimization (PMOO) method is shown in Figure 1 [12]. In Figure 1, k indicates the total number of the candidate alternatives in the relevant system; l represents the total number of objective response indexes of each alternative; $Y_{\alpha\beta}$ is the utility value of the β-th objective index of the α-th alternative; $P_{\alpha\beta}$ represents the partial preferable probability of the beneficial attribute index$Y_{\alpha\beta}$; A_β represents the normalization factor of the β-th beneficial attribute (objective) indicator, $A_\beta = 1/(k\overline{Y_\beta})$. in which $\overline{Y_\beta}$ is the average utility value of the β-th attribute index in the attribute group involved, $\overline{Y}_\beta = \left(\sum_{\alpha=1}^{k} Y_{\alpha\beta}\right)/k$: both $Y_{\beta\min}$ and $Y_{\beta\max}$ represent the minimum and maximum utility values of the attribute indexes in the β-th objective (attribute) group, respectively; B_β is the normalization factor of the β-th unbeneficial attribute index with $B_\beta = 1/\left[k(Y_{\beta\,\max} + Y_{\beta\,\min}) - k\overline{Y}_\beta\right]$.

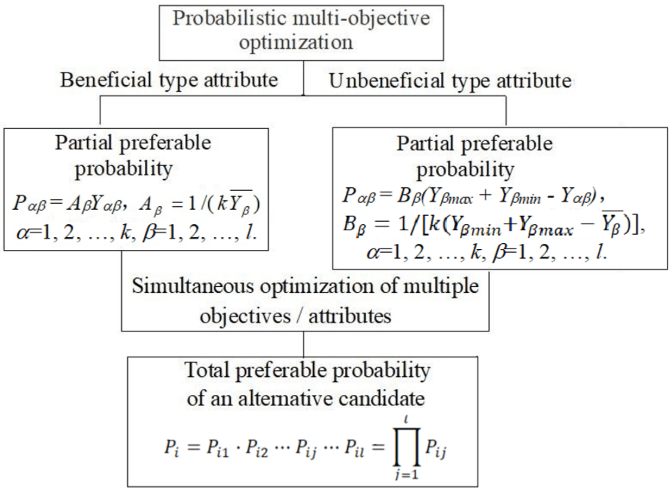

Figure 1 Process of probabilistic multi-objective optimization method

3 Solution to the optimization of aromatic extraction process

The parametric optimization of aromatic extraction process is to provide a set of specific (optimum) parameters to ensure maximizing the overall efficiency of the aromatics unit system and minimizing process energy consumption at the same time, so PMOO approach might be the proper choice.

Chen B. et al used ASPEN Plus's module to conduct a simulation, and the optimal variables include the top circulation of stripping tower x_1, the steam flow of solvent recovery tower x_2, and the top circulation of benzene tower x_3, which are utilized as the operating variables in the optimization process. Besides, response surface method is employed to conduct analysis of simulation data [13], the results are shown in Table 1. Now this problem is restudied by using probabilistic multi-objective optimization (PMOO) together with regression.

Taking the maximization of product purity η (beneficial type of attribute) and the minimization of process energy consumption E (unbeneficial or cost type of attribute) as the optimal objectives, the evaluation is conducted according to probabilistic multi-objective optimization (PMOO), the results are obtained and shown in Table 2, where P_η is the preferable probability of product purity η, P_E is the preferable probability of energy consumption E, and P_i is the total preferable probability of the corresponding scheme.

Table 1 Simulation test results

No.	Operating variable						Production	
	Real data, kmol/h			Code			Purity, %	Energy consumption, kw
	x1	x2	x3	x1	x2	x3	η	E
1	63	4.9	1.1	-1	-1	0	0.99654	3390.396
2	63	4.9	1.3	-1	-1	1	0.99768	3558.144
3	63	7.1	1.1	-1	0	0	0.99981	3374.801
4	63	7.1	1.3	-1	0	1	0.99989	3543.631
5	**63**	**9.3**	**1.1**	**-1**	**1**	**0**	**0.99681**	**3359.554**
6	63	9.3	1.3	-1	1	1	0.99664	3528.910
7	68	4.9	1.1	0	-1	0	0.99934	3456.554
8	68	4.9	1.3	0	-1	1	0.99957	3623.675
9	68	7.1	1.1	0	0	0	0.99983	3439.727
10	68	7.1	1.3	0	0	1	0.99991	3607.347
11	**68**	**9.3**	0.9	**0**	**1**	**-1**	**0.98521**	**3257.160**
12	68	9.3	1.1	0	1	0	0.99683	3424.318
13	68	9.3	1.3	0	1	1	0.99653	3595.934
14	73	4.9	1.1	1	-1	0	0.99973	3521.808
15	73	7.1	1.1	1	0	0	0.99984	3503.543
16	73	9.3	0.9	1	1	-1	0.98510	3318.362
17	73	9.3	1.1	1	1	1	0.99685	3486.583
18	73	9.3	1.3	1	1	1	0.99679	3655.879

Table 2 The results of evaluation according to probabilistic multi-objective optimization (PMOO)

No.	Purity, %	Energy Consumption kw	Partial Preferable Probability		Total Preferable Probability	Rank
	η	E	P_η	P_E	$P_i \times 103$	
1	0.99654	3390.396	0.0555	0.057	3.1664	5
2	0.99768	3558.144	0.0556	0.0543	3.019	14
3	0.99981	3374.801	0.0557	0.0573	3.1909	4
4	0.99989	3543.631	0.0557	0.0545	3.0388	13
5	0.99681	3359.554	0.0556	0.0575	3.195	2
6	0.99664	3528.91	0.0555	0.0548	3.0422	12
7	0.99934	3456.554	0.0557	0.0559	3.1156	8
8	0.99957	3623.675	0.0557	0.0532	2.9657	17
9	0.99983	3439.727	0.0557	0.0562	3.1326	7
10	0.99991	3607.347	0.0557	0.0535	2.9814	16
11	0.98521	3257.16	0.0549	0.0592	3.2488	1
12	0.99683	3424.318	0.0556	0.0565	3.1368	6
13	0.99653	3595.934	0.0555	0.0537	2.9816	15
14	0.99973	3521.808	0.0557	0.0549	3.058	11
15	0.99984	3503.543	0.0557	0.0552	3.0748	10
16	0.9851	3318.362	0.0549	0.0582	3.194	3
17	0.99685	3486.583	0.0556	0.0555	3.0809	9
18	0.99679	3655.879	0.0556	0.0527	2.9285	18

As can be seen from Table 2, the primary optimized result is the 11^{-th} scheme, which is followed by the 5^{-th}. Therefore, the optimal configuration is not far from the parameters of the 11^{-th} scheme. The parameters of the 11^{-th} scheme (x_1' = 68 kmol/h, x_2' = 9.3 kmol/h and x_3' = 0.9 kmol/h) can be regarded as the first-order approximation, and the corresponding purity η' and energy consumption E' are 98.5210% and 3257.1600kw, respectively.

Furthermore, regression could be employed to conduct further optimization. The functional relationship between the total preferable probability P_i and x_1, x_2 and x_3 can be regressed first, and the optimal allocation under the second-order approximation can be obtained by finding the extreme value of the function of the total preferable probability P_i with respect to the optimal variables x_1, x_2 and x_3. The regression results are shown in Eq. (1) through Eq. (3).

$$\begin{aligned}\mathrm{P_i} \times 10^3 =& 3.914625 - 0.01418x_1 + 0.041766x_2 + 0.49422x_3 + 2.64 \times 10^{-5}x_l^2 \\ & - 0.00153x_2^2 - 0.50695x_3^2 - 0.00012x_lx_2 - 0.00604x_2x_3 \\ & + 0.000176x_3x_l, \quad \mathrm{R}^2 = 0.99979\end{aligned} \tag{1}$$

$$\begin{aligned}\eta =&0.698306 + 0.001297x_1 + 0.011675x_2 + 0.357314x_3 - 5.7 \times 10^{-6}x_1^2\\&- 0.00045x_2^2 - 0.14673x_3^2 - 6.6 \times 10^{-5}x_1x_2 - 0.00106x_2x_3\\&+ 5.77 \times 10^{-5}x_3x_1, \quad \mathrm{R}^2 = 0.99365\end{aligned} \tag{2}$$

$$\begin{aligned}\mathrm{E} =&1491.581 + 20.12879x_1 - 7.4583x_2 + 762.4528x_3 - 0.04749{x_l}^2 - 0.151715x_2^2\\&+ 27.26057x_3^2 - 0.08834x_1x_2 + 3.512453x_2x_3 - 0.1594x_3x_1, \quad \mathrm{R}^2 = 0.99997\end{aligned} \tag{3}$$

It could obtain the optimum results of second-order approximation with the configuration parameters of x_1^* = 63.8261 kmol/h, x_2^* = 6.9022 kmol/h, x_3^* = 0.9065 kmol/h, $P_t \times 10^3$ = 3.2855, and optimal product purity of η^* = 98.7898% and energy consumption of E^* = 3226.0453 kw. Obviously, the optimal product purity of η^* and energy consumption of E^* are much better than those of η' and E' from first-order approximation, *i.e.*, primary evaluation of test data.

4 Discussion

4.1 Extensibility

Under condition of multi-objective optimization with complexity, such as many objectives and variables (more than 6 for example), experimental design methods, *i.e.*, orthogonal experimental design, response experimental design and uniform experimental design, etc., can be employed to perform simplicity.

4.2 Advantage of PMOO

The obvious advantages of PMOO include: a) the exact / clear definition of “optimum point” in multi-objective optimization problem in a system; b) no need to manually set weight factors.

5 Summary

From the above discussion, it obtains following conclusions.

(1) The optimization of aromatic extraction process is conducted by means of probabilistic multi-objective optimization, which stresses the simultaneity of multiple objectives in the optimization in terms of preferable probability.

(2) The obvious feature of probabilistic multi-objective optimization is without any artificial or subjective scaling factor, which opens up a new way to solve multi-objective problems and has broad application prospects.

(3) Regression is an effective way to get the optimum status and corresponding parameters with limited reliable test data.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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