Res Intell Manuf Assem, 2026, 5(1): 313-320
DOI: 10.25082/RIMA.2026.01.002

RESEARCH ARTICLE

Solution of Multi-objective Management Problem by Means of Probabilistic Multi-objective Optimization Approach

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Received: October 9, 2025;
Accepted: January 8, 2026;
Published: January 15, 2026.

Citation: Zheng M, Yu J. Solution of Multi-objective Management Problem by Means of Probabilistic Multi-objective Optimization Approach. *Res Intell Manuf Assem*, 2026, **5**(1): 313-320.
https://doi.org/10.25082/RIMA.2026.01.002

Abstract: This paper presents the application of probabilistic multi-objective optimization method (PMOO) in enterprise production management, which involves the simultaneous optimization of "high long-term profit target" and "small investment amount". PMOO method is an effective approach to deal with multi-objective optimization problems from the viewpoint of system theory and method of probability theory, in which the new concept of "preferable probability" is introduced to formulate the methodology of PMOO. In PMOO, the evaluated attributes (objectives) of candidates are preliminarily divided into two basic types: beneficial attributes and unbeneficial attributes, and the corresponding quantitative evaluation method of partial preferable probability of each type of attribute is established. Furthermore, the total preferable probability of each candidate alternative is the product of partial preferable probabilities of all possible attributes, and the maximum value of the total preferable probability presents the overall optimization of the system. In the enterprise production management problem of three kinds of products, the objective function is to maximize the long-term profit target and minimize the investment amount, the discretization of Hua's "good lattice point" and uniform mixture design are applied to simplify the optimization process and data processing. Finally, a rational result is obtained.

Keywords: probabilistic multi-objective optimization, preferable probability, target management, uniform design, discretization

1 Introduction

Target management and planning are derived from linear programming in general. In 1961, A. B. Charnes and W. W. Cooper put forward relevant concepts and models when considering the approximate solution of infeasible linear programming problems [1]. At present, some solutions have been developed, each is with its own advantages. For the optimization (option) problem with multiple attributes in a system, the optimization criteria of each attribute may usually be contradictory. Such problems belong to "multi-objective optimization problems" and need to be solved by "multi-objective optimization methods" generally.

At present, some multi-objective optimization methods have been developed, such as, simple additive weighting (SAW) [2], weighted aggregation and product evaluation (WASPAS) [3], preference method similar to ideal solution (TOPSIS) [4], VIKOR method (VLSEKriterijumska optimizacija i komprominisno Resenje) [5], analytic hierarchy process (AHP), multi-objective optimization method based on ratio analysis (MORA) [6], compound proportional evaluation (COPRAS) [7], neighborhood index value (PIV) [8], preference selection index (PSI) [9], preference selection index (PSIE) determined by entropy method [10], etc. These methods have been applied in many fields to varying degrees. However, there are essential deficiencies in the above methods. For example, in the linear weighting method, if the objective functions $f_1(x)$, $f_2(x)$, ..., $f_p(x)$ are "added" with the weight coefficient w_j, there is no objectivity in the selection of the weight coefficient, and each attribute needs to be "normalized" when the dimensions of the objective are different, and the selection of the normalized denominator is "each needs what he wants", which is lack of rationality. Some methods also introduce artificial factors such as virtual "ideal point". Not only that, generally speaking, in set theory, "addition" is union; in probability theory, "addition" is the "sum" of events. Therefore, it can be seen that the operation mode of "addition" fundamentally deviates from the original intention of "simultaneous optimization" of the multi-objective optimization. Pareto solution can only give a set of solutions. These cases indicate that the method of multi-objective (attribute) optimization is not perfect.

In fact, the original intention of multi-objective optimization (optimization) is to "simultaneously optimize multiple objectives" in a system. From the perspective of probability theory, it is the "product" of probability of each objective; in set theory, it belongs to the "intersection" of various objectives.

In view of the above situation, in recent years, from the viewpoint of system theory, we regard multi-objective optimization as a problem of "simultaneous optimization" of multiple objectives in a system. Therefore, the "optimal point of multiple objectives" in a system is the "optimum point of the whole system"; furthermore, when using the methods of set theory and probability theory to deal with this multi-objective optimization problem, the new concept of "preferable probability" is introduced to reflect the preference degree of the objective in the optimization, and the theory and method system of probabilistic multi-objective optimization (PMOO) is established, which is a probabilistic multi-objective optimization method [10]. In PMOO, the evaluation objectives (attributes) of the candidates in the optimization task can be divided into two basic types: beneficial attribute and unbeneficial (or cost) attribute, and a set of quantitative evaluation methods of the relative preferable probabilities for both beneficial attribute and unbeneficial attribute is established [10]. Usually, the whole optimization is regarded as a system, and "multiple attributes" are optimized at the same time, which is analogical to the problem of "multiple events appearing at the same time" in probability theory. Therefore, the total preferable probability of each candidate is the product of partial preferable probabilities of all possible attributes of the candidate object. Thus the overall optimization of the system can be handled. Finally, all the candidate alternatives are ranked according to their total preferable probability, which is the unique and decisive index for the candidate alternative to win the competition in this optimization. In a word, further analysis shows that the probabilistic multi-objective optimization method is obviously different from other methods. This is mainly reflected in the fact that "probabilistic multi-objective optimization" has both viewpoints and methods. According to the viewpoint of system theory, it is concluded that "the optimal point of multi-objective optimization" is "the optimum point of the system", and then this optimum point of the system is obtained by probability theory. However, other "multi-objective optimization methods" in the past have "only methods without opinions", that is, what is the optimal point of "multi-objective optimization" is not defined.

In this paper, a probabilistic multi-objective optimization method is utilized to solve the multi-objective management problem in order to establish a more effective method. Specifically, the PMOO and the uniform design method of mixture are adopted to implement the optimization.

2 A Brief Introduction of Probabilistic Multi-objective Optimization Method

As mentioned earlier, the original intention of multi-objective optimization is to "simultaneously optimize" multiple objectives in a system. From the perspective of probability theory, it is the "product" of the probability of each objective and the "intersection" of each attribute in set theory. Our method is to handle this problem by making an analogy with the problem of "multiple events appearing at the same time" in probability theory, it introduces the new concept of "preferable probability", which is used to reflect the preference degree of the objective in the optimization. Furthermore, the evaluated objectives (attributes) of candidates in the optimization task are preliminarily divided into two basic types: beneficial attribute and unbeneficial (or cost) attribute, and a quantitative evaluation of partial preferable probabilities are as follows: the partial preferable probability

P_{ij} of a beneficial attribute is linearly related to the utility Y_{ij} of the corresponding attribute positively, and the partial preferable probability P_{ij} of an unbeneficial attribute is linearly related to the utility Y_{ij} of the corresponding attribute negatively [10]. Therefore, the total preferable probability P_i of each candidate alternative is the product of partial preferable probabilities P_{ij} of all possible attributes of the candidate alternative. Finally, a ranking could be conducted according to the value of the total preferable probability of each candidate alternative, the one with the maximum total preferable probability win the competition in this optimization.

As to the concept of "preferable probability", it reflects the preference degree of the objective in the optimization. It is something like the "score of beauty contest". The judges graded each player according to his appearance, talent and other indicators (goals). Finally, we convert all the judges' scores into a comprehensive score through a rule and normalize it to a percentage form. For example, "player A's preferable probability is a some value (says 80% for example)" does not mean that he/she is likely to be an individual exactly at the corresponding value, but means that under the current judges and grading standards, he/she is more preferable than other players. This probability value is especially constructed for ranking and decision-making. However, Jaynes's probability is like the "weather forecast". The Meteorological Observatory calculated that "the probability of rain tomorrow is at a certain value (says 45% for example)" according to historical data and models. This kind of probability is an estimation of the uncertainty of the objective natural phenomenon (rain). It does not include any subjective preference of "I like rain" or "rain is better". Therefore, the concept of "preferable probability" is a new idea to indicate the preference degree of the objective in the optimization and contest for decision-making particularly.

Why does the "optimum point" of multi-objective always favor the maximum probability? This is due to the definition of preferable probability! Many phenomena in nature take the maximum value, from relativity to quantum mechanics, optics, electromagnetism, etc. In order to describe the related phenomena, principle of least action was specially put forward. The process of probabilistic multi-objective optimization method is shown in Figure 1 [10]. Besides, the details of regulations in PMOO evaluation are as follows:

(1) Evaluation of partial preferable probability in case of beneficial type (the bigger the better) of attribute is expressed in Equation (1) [10],

$$P_{ij} = \alpha_j Y_{ij},\ \alpha_j = 1/(k\overline{Y_j}),\ i\ = 1, 2, \ldots, k;\ \ j = 1, 2, \ldots, l. \tag{1}$$

(2) Evaluation of partial preferable probability in case unbeneficial type (the smaller the better) of attribute is presented in Equation (2) [10],

$$\begin{aligned} P_{ij} &= \beta_j(Y_{j\max} + Y_{j\min} - Y_{ij}),\\ \beta_j &= 1/[k(Y_{j\max} + Y_{j\min} - \overline{Y_j})],\\ i &= 1, 2, \ldots, k;\ j = 1, 2, \ldots, l. \end{aligned} \tag{2}$$

(3) Total preferable probability of an alternative candidate is given in Equation (3) [10],

$$P_i = P_{i1} \cdot P_{i2} \cdots P_{il} = \prod_{j=1}^{l} P_{ij},\ i\ = 1, 2, \ldots, k;\ \ j = 1, 2, \ldots, l. \tag{3}$$

In Equation (1) through Equation (3), P_{ij}reflects the preferable probability of the *j-th* performance attribute of the *i-th* alternative [10], k is the total number of alternative candidates, and l is the total number of performance attributes; P_i indicates the total preferable probability of the *i-th* candidate; Y_{ij} reflects the utility index value of the *j-th* performance attribute of the *i-th* alternative candidate; α_j is the normalization factor of the *j-th* beneficial type of performance attributes index; β_j reflects the normalization factor of the *j-th* unbeneficial type of performance attribute index; $\overline{Y_j}$ is the arithmetic average value of *j-th* utility index of the performance attribute in the evaluated group; Y_{jmin} and Y_{jmax} represent the minimum and maximum values of the utility index Y_{ij} of the *j-th* objective in the evaluated group, respectively.

Besides, the evaluations of normalization factors α_j and β_j are obtained from the general principle of normalization of probability theory for P_{ij} over all alternative candidates i [10], *i.e.*,

$$\sum\nolimits_{i=1}^{k} P_{ij} = \sum\nolimits_{i=1}^{k} \alpha_j Y_{ij} = 1 \tag{4}$$

and

$$\sum\nolimits_{i=1}^{k} P_{ij} = \sum\nolimits_{i=1}^{k} \beta_j(Y_{j\max} + Y_{j\min} - Y_{ij}) = 1 \tag{5}$$

Thus, it leads to the following results for normalization factors α_j and β_j [10],

$$\alpha_j = 1/(k\overline{Y_j}) \tag{6}$$

and

$$\beta_j = 1/[k(Y_{j\max} + Y_{j\min} - \overline{Y_j}) \tag{7}$$

Compared with other methods, the probabilistic multi-objective optimization method has fundamental differences, as shown in Table 1.

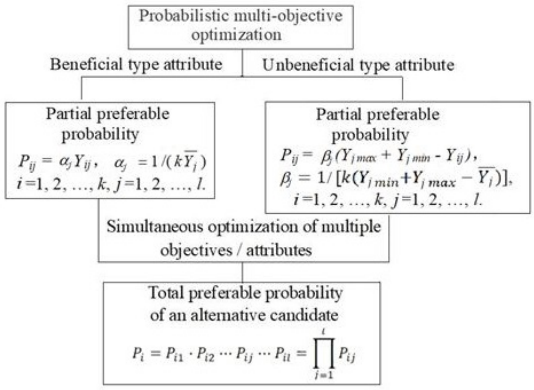

Figure 1 Procedure of PMOO assessment

Table 1 Comparison of PMOO with respect to other methods

Method / Aspect	PMOO	Other methods
Opinion	System theory.	No.
Optimum point	Maximizing total preferable probability.	Not defined.
Characteristic	As a system, the whole system is optimized.	Compromise.
Quantitative method	Probability method.	Normalization.
Feature of algorithm	Total preferable probability equals to product of all- -possible partial preferable probabilities.	Addition of normalized objectives.
Characteristic in set theory	Intersection.	Union.
Characteristic of solution	Unique.	Un-unique.
Subjective factor	No.	Weighting factor, normalized denominator, etc.
Cost	Lower.	Higher.
Combination with design of experiment	Yes.	No.
Trapped in local optimum	No.	Easily.
Robust design	Treat both μ and σ equally, simultaneously and separately.	Combine μ and σ into "signal to noise ratio".

3 Solution of a Multi-objective Manage Problem

As an application example, a multi-objective management problem is dealt with to illuminate the procedure and to show the operation process.

The problem is described as following. There are three kinds of products to be produced by a company: says *A*, *B* and *C*. Let x_1, x_2 and x_3 represent the output of these three products respectively. The optimal objectives of this problem are: 1) the long-term profit target is not less than 125 million yuan; 2) The investment target shall not exceed 79 million yuan. Moreover, the number of employees has remained at around 4,000. The relevant parameters of the product are given in Table 2.

Table 2 The relevant parameters of the products

Factor	Contribution of each piece of product			Objective or constraint
	A	B	C	
long-term profit / million yuan	12	9	15	≥ 125
Number of employee / hundred	5	3	4	$\cong 40$
Investment / million yuan	5	7	8	≤ 79

From the meaning of the problem and Table 2, the following relationship can be obtained,

$$Max\ f_1 = 12x_1 + 9x_2 + 15x_3 - 125 \geq 0; \tag{8}$$

$$Max\ f_2 = 79 - 5x_1 - 7x_2 - 8x_3 \geq 0; \tag{9}$$

$$s.t.: 5x_1 + 3x_2 + 4x_3 = 40; \tag{10}$$

$$x_1,\ x_2,\ x_3 \geq 0. \tag{11}$$

The actual range of the independent variables x_1, x_2, and x_3 is, $0 \leq x_1 \leq 8, 0 \leq x_2 \leq 40/3$, $0 \leq x_3 \leq 10$.

Let y_1 = *0.125x*$_1$, y_2 = *3x*$_2$*/40*, y_3 = *x*$_3$*/10*, then the above formula and conditions can be reduced to following forms,

$$Max\ f_1 = 96y_1 + 120y_2 + 150y_3 - 125 \geq 0; \tag{12}$$

$$Max\ f_2 = 79 - 40y_1 - 280y_2/3 - 80y_3 \geq 0; \tag{13}$$

$$s.t.: y_1 + y_2 + y_3 \cong 1; \tag{14}$$

$$y_1,\ y_2,\ y_3 \geq 0. \tag{15}$$

Therefore, the actual range of y_1, y_2, and y_3 is, $0 \leq y_1 \leq 1$, $0 \leq y_2 \leq 1$, $0 \leq y_3 \leq 1$. This is a bi-objective optimization problem, and because of the constraint condition $y_1 + y_2 + y_3 = 1$, it actually contains only two independent variables, namely y_1 and y_2. We can choose to use the uniform mixing test design for processing [11, 12]. Because the sampling points need to be laid out in three-dimensional space, it is necessary to ensure that at least 19 uniform test sampling points are included in the effective area. Uniform experimental design is a method system founded by Prof. Fang Kaitai and Prof. Wang Yuan [11, 12]. This method is based on the approximate calculation of "good lattice points" adopted by Prof. Hua Luogeng in his early years, and the research results with good convergence are obtained [13, 14], thus making uniform distribution a more classical discretization method. Use $U_{89}(89)$ to construct a uniform design table $UM_{89}(89^3)$ for mixing materials, as shown in Table 3.

Table 3 Design table $UM_{89}(89^3)$ on basis of uniform experimental table $U_{89}(89)$

No.	y10	y20	c1	c2	y1	y2	y3	No.	y10	y20	c1	c2	y1	y2	y3
1	1	9	0.0056	0.0955	0.9250	0.0678	0.0072	46	46	58	0.5112	0.6461	0.2850	0.2531	0.4619
2	2	18	0.0169	0.1966	0.8702	0.1043	0.0255	47	47	67	0.5225	0.7472	0.2772	0.1827	0.5401
3	3	27	0.0281	0.2978	0.8324	0.1177	0.0499	48	48	76	0.5337	0.8483	0.2694	0.1108	0.6197
4	4	36	0.0393	0.3989	0.8017	0.1192	0.0791	**49**	**49**	**85**	**0.5449**	**0.9494**	**0.2620**	**0.0373**	**0.7009**
5	5	45	0.0506	0.5000	0.7751	0.1124	0.1124	50	50	5	0.5562	0.0506	0.2542	0.7081	0.0377
6	6	54	0.0618	0.6011	0.7514	0.0992	0.1494	51	51	14	0.5674	0.1517	0.2467	0.6390	0.1143
7	7	63	0.0730	0.7022	0.7298	0.0805	0.1898	52	52	23	0.5789	0.2528	0.2393	0.5684	0.1923
8	8	72	0.0843	0.8034	0.7097	0.0571	0.2332	53	53	32	0.5899	0.3539	0.2320	0.4962	0.2718
9	9	81	0.0955	0.9045	0.6910	0.0295	0.2795	54	54	41	0.6011	0.4551	0.2247	0.4225	0.3528
10	10	1	0.1067	0.0056	0.6733	0.3249	0.0018	55	55	50	0.6124	0.5562	0.2175	0.3473	0.4352
11	11	10	0.1180	0.1067	0.6565	0.3068	0.0367	56	56	59	0.6236	0.6573	0.2103	0.2706	0.5191
12	12	19	0.1292	0.2079	0.6405	0.2847	0.0747	57	57	68	0.6348	0.7584	0.2032	0.1925	0.6043
13	13	28	0.1404	0.3090	0.6252	0.2590	0.1158	58	58	77	0.6461	0.8596	0.1962	0.1129	0.6909
14	14	37	0.1517	0.4101	0.6105	0.2297	0.1597	**59**	**59**	**86**	**0.6573**	**0.9607**	**0.1893**	**0.0319**	**0.7789**
15	15	46	0.1629	0.5112	0.5964	0.1973	0.2064	60	60	6	0.6685	0.0618	0.1824	0.7671	0.0505
16	16	55	0.1742	0.6124	0.5827	0.1618	0.2556	61	61	15	0.6798	0.1629	0.1755	0.6902	0.1343
17	17	64	0.1854	0.7135	0.5694	0.1234	0.3072	62	62	24	0.6910	0.2640	0.1687	0.6118	0.2195
18	18	73	0.1966	0.8146	0.5566	0.0822	0.3612	63	63	33	0.7022	0.3652	0.1620	0.5320	0.3060
19	19	82	0.2079	0.9157	0.5441	0.0384	0.4175	64	64	42	0.7135	0.4663	0.1554	0.4509	0.3939
20	20	2	0.2191	0.0169	0.5319	0.4602	0.0079	65	65	51	0.7247	0.5674	0.1487	0.3683	0.4830
21	21	11	0.2303	0.1180	0.5201	0.4233	0.0566	66	66	60	0.7360	0.6685	0.1421	0.2844	0.5735
22	22	20	0.2416	0.2191	0.5085	0.3838	0.1077	67	67	69	0.7472	0.7697	0.1356	0.1991	0.6653
23	23	29	0.2528	0.3202	0.4972	0.3418	0.1610	68	68	78	0.7584	0.8709	0.1291	0.1125	0.7583
24	24	38	0.2640	0.4213	0.4861	0.2973	0.2165	69	69	87	0.7697	0.9719	0.1227	0.0246	0.8527
25	25	47	0.2753	0.5225	0.4753	0.2505	0.2741	70	70	7	0.7809	0.0730	0.1163	0.8191	0.0645
26	26	56	0.2865	0.6236	0.4647	0.2015	0.3338	71	71	16	0.7921	0.1742	0.1100	0.7350	0.1550
27	27	65	0.2978	0.7247	0.4543	0.1502	0.3955	72	72	25	0.8034	0.2753	0.1037	0.6496	0.2467
28	28	74	0.3090	0.8258	0.4441	0.0968	0.4591	73	73	34	0.8146	0.3764	0.0974	0.5628	0.3397
29	29	83	0.3202	0.9270	0.4341	0.0413	0.5246	74	74	43	0.8258	0.4775	0.0912	0.4748	0.4340
30	30	3	0.3315	0.0281	0.4243	0.5596	0.0162	75	75	52	0.8371	0.5787	0.0851	0.3855	0.5294
31	31	12	0.3427	0.1292	0.4146	0.5098	0.0756	76	76	61	0.8483	0.6798	0.0790	0.2949	0.6261
32	32	21	0.3539	0.2303	0.4051	0.4579	0.1370	77	77	70	0.8596	0.7809	0.0729	0.2031	0.7240
33	33	30	0.3652	0.3315	0.3957	0.4040	0.2003	78	78	79	0.8708	0.8820	0.0668	0.1101	0.8231
34	34	39	0.3764	0.4326	0.3865	0.3481	0.2654	79	79	88	0.8820	0.9831	0.0608	0.0158	0.9233
35	35	48	0.3876	0.5337	0.3774	0.2903	0.3323	80	80	8	0.8933	0.0843	0.0549	0.8658	0.0796
36	36	57	0.3989	0.6348	0.3684	0.2306	0.4009	81	81	17	0.9045	0.1854	0.0490	0.7747	0.1763
37	37	66	0.4101	0.7360	0.3596	0.1691	0.4713	82	82	26	0.9157	0.2865	0.0431	0.6828	0.2742
38	38	75	0.4213	0.8371	0.3509	0.1058	0.5434	83	83	35	0.9270	0.3876	0.0372	0.5896	0.3732
39	39	84	0.4326	0.9382	0.3423	0.0406	0.6171	84	84	44	0.9382	0.4888	0.0314	0.4952	0.4734
40	40	4	0.4438	0.0393	0.3338	0.6400	0.0262	85	85	53	0.9495	0.5899	0.0257	0.3996	0.5748
41	41	13	0.4551	0.1404	0.3254	0.5798	0.0947	86	86	62	0.9607	0.6910	0.0199	0.3029	0.6773
42	42	22	0.4663	0.2416	0.3172	0.5179	0.1650	87	87	71	0.9719	0.7921	0.0141	0.2049	0.7809
43	43	31	0.4775	0.3427	0.3090	0.4542	0.2368	88	88	80	0.9831	0.8933	0.0085	0.1058	0.8857
44	44	40	0.4888	0.4438	0.3009	0.3888	0.3103	89	89	89	0.9944	0.9944	0.0028	0.0056	0.9916
45	45	49	0.5000	0.5449	0.2929	0.3218	0.3853								

The specific implementation steps are as follows:

(1) Selecting the uniform design table for given the number of independent variables s and number of effective sampling points n. The number of independent variables s is 3 in this topic, and the number of effective sampling points n is at least 19 in this topic, the corresponding table $U*_n(n^t)$ or $U_n(n^t)$ and usage table [11, 12] are selected from the uniform design table provided by Prof. Fang Kaitai, and the number of columns in the usage table is selected as *s-1*. And mark the original elements in the uniform design table $U*_n(n^t)$ or $U_n(n^t)$ with $\{q_{ik}\}$.

(2) Constructing a new element c_{ki} for each i. The construction of c_{ki} is according to the following formula, $c_{ki} = (2q_{ki} - 1)/(2n)$.

(3) Constructing a uniform sampling point y_{ki} of the mixture. The construction of a uniform sampling point y_{ki} of the mixture is according to the following formula, $y_{ki} = (1 - {c_{ki}}^{\frac{1}{s-i}}) \prod_{j=1}^{i-1} {c_{kj}}^{\frac{1}{s-j}}$, $i = 1, \ldots, s-1$. $y_{ks} = \prod_{j=1}^{s-1} {c_{kj}}^{\frac{1}{s-j}}$, $k = 1, \ldots, n$.

Then, $\{y_{ik}\}$ gives the corresponding uniform design table $UM_n(n^s)$ for mixture under s and n conditions.

The uniform design table $UM_{89}(89^3)$ in Table 3 is based on $U_{89}(89)$, and y_{10} and y_{20} are the original coordinates of sampling points in the [1, 89]×[1, 89] area. Since s = 3 and n = 19 here, according to the above rule, the results of functions f_1 and f_2 at discrete points can be then obtained, among which 21 sampling points satisfy the conditions of $f_1 \geq 0$ and $f_2 \geq 0$. See Table 4 for the distribution of preferable probability and ranking of functions f_1 and f_2 at discrete points and sampling points. Figure 2 is the projection of effective sampling points on x_1–x_2 and x_2–x_3 planes. The results show that the discretized 49-th sampling point gives the maximum total preferable probability, and 59-th sampling point is the next, so they can be used as the optimal solution of this bi-objective optimization problem.

Table 4 Data of functions f_1 and f_2 on sampling points, and evaluation results of their preferable probabilities

No.	f_1	f_2	P_{f1}	P_{f2}	$P_i \times 10^3$	Rank
29	0.3196	15.8133	0.0019	0.1337	0.2482	18
37	0.5086	11.1293	0.0030	0.0941	0.2779	17
38	2.8924	11.6173	0.0168	0.0982	1.6499	10
39	5.2978	12.1507	0.0308	0.1027	3.1607	3
46	2.0170	7.0253	0.0117	0.0594	0.6958	14
47	4.5502	7.6520	0.0264	0.0647	1.7096	9
48	7.1134	8.3067	0.0413	0.0702	2.9013	5
49	**9.7630**	**8.9667**	**0.0567**	**0.0758**	**4.2984**	**1**
54	0.1912	2.3547	0.0011	0.0199	0.0221	20
55	2.8360	3.0693	0.0165	0.0259	0.4274	15
56	5.5258	3.8040	0.0321	0.0322	1.0321	12
57	8.2522	4.5613	0.0479	0.0386	1.8482	8
58	11.0182	5.3427	0.0640	0.0452	2.8904	6
59	**13.8358**	**6.1387**	**0.0804**	**0.0519**	**4.1703**	**2**
65	5.9212	0.0373	0.0344	0.0003	0.0109	21
66	8.7946	0.8920	0.0511	0.0075	0.3852	16
67	11.7046	1.7693	0.0680	0.0150	1.0168	13
68	14.6386	2.6720	0.0850	0.0226	1.9206	7
69	17.6362	3.5800	0.1025	0.0303	3.1001	4
78	18.0898	0.2040	0.1051	0.0017	0.1812	19
79	21.2278	1.2293	0.1233	0.0104	1.2813	11

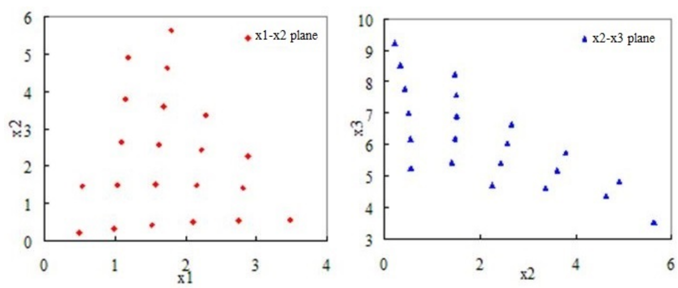

Figure 2 Projection of effective sampling points on x_1–x_2 and x_2–x_3 planes. a) Projection of effective sampling points on the x_1–x_2 plane, b) Projection of effective sampling points on the x_2–x_3 plane

For the 49-th sampling point, the independent variables are $x_1* = 2.0960$, $x_2* = 0.4973$, $x_3* = 7.0090$, it results that f_1 is 9.7630 and f_2 is 8.9667, that is, the long-term profit is 134.7630 million yuan, and the investment value is 70.0333 million yuan. After rounding, take $x_1{}^*=2$, $x_2{}^*=1$, $x_3{}^*=7$, it obtains $f_1= 13$, and $f_2= 6$, while the long-term profit is 138 million yuan, the investment value is 74 million yuan, and the number of employees has remained at around 4100.

As to the 59-th sampling point, the independent variables are $x_1* = 1.5144$, $x_2* = 0.4253$, $x_3* = 7.7890$, it obtained that f_1 is 13.8358, and f_2 is 6.1387, that is, the long-term profit is 138.8358 million yuan, and the investment value is 72.8613 million yuan. After rounding, take $x_1* = 2$, $x_2* = 0$ and $x_3* = 8$, it gives f_1 of 19 and f_2 of 5, that is, the long-term profit is 144 million yuan, the investment value is 75 million yuan, and it can accommodate 4,200 people.

4 Summary

From above study, it can be obtained following realization preliminarily: 1) PMOO could be used to solve multi-objective management problem. It characterizes the simultaneity of optimization of multiple objectives in the multi-objective optimization process. It reveals and strengths the irreplacibility of each objective in the multi-objective optimization process; 2) It avoids subjective factors such as weighting factors in previous methods; 3) It initiates a new way for solving multi-objective problems and has broad application prospects.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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