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RESEARCH ARTICLE

Physics-Informed Digital Twin for Robust Inverse Parameter Identification under Sparse and Noisy Measurements

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Abstract: Reliable inverse identification of mechanical system parameters from sparse and noisy measurements remains a major challenge for digital twin applications because conventional identification methods are highly sensitive to measurement uncertainty and limited sensing data. This study presents a physics-informed digital twin framework that integrates the governing equation of motion with a neural network to simultaneously reconstruct system dynamics and estimate unknown mechanical parameters while maintaining physical consistency. The framework is developed for a single-degree-of-freedom mass–spring–damper system and jointly optimizes the displacement response together with the mass, damping, and stiffness using a two-stage Adam–L-BFGS optimization strategy. Its performance is evaluated through convergence analysis, basin stability, structural identifiability, ensemble-based uncertainty quantification, comparisons with least-squares, recursive least-squares, and data-driven neural networks, validation using independent literature-based parameter sets, and sensitivity analysis under signal-to-noise ratios ranging from 5 to 40 dB. The proposed framework reconstructs the system response with RMSE values of 0.013–0.021 while identifying mass, damping, and stiffness with relative errors of approximately 1.1%, 1.1%, and 1.3%, respectively. Compared with conventional and purely data-driven approaches, it achieves more accurate parameter estimation, lower reconstruction error (RMSE = 0.016), consistent convergence across multiple random initializations, and reliable 95% confidence intervals for prediction uncertainty. Unlike previous studies that primarily demonstrate inverse parameter recovery using physics-informed neural networks, this work provides a comprehensive reliability-oriented evaluation by integrating optimization robustness, structural identifiability, uncertainty quantification, comparative benchmarking, and multi-noise validation within a single physics-informed digital twin framework, improving the reliability and interpretability of inverse parameter identification under limited sensing conditions.

Keywords: physics-informed neural networks, digital twin, inverse problems, system identification, uncertainty quantification, Mechanical dynamics

1 Background

Classical system identification methods, including frequency-domain techniques, time-domain regression, least-squares estimation, and state-space approaches, have been widely used to estimate the physical parameters of mechanical systems. Although effective with dense, high-quality measurements, their performance deteriorates under sparse and noisy observations, where inverse parameter estimation becomes ill-posed and highly sensitive to measurement uncertainty. Conventional methods therefore often require filtering or regularization that may compromise physical fidelity. Recent machine learning approaches have improved predictive capability but generally require large datasets and may violate governing physical laws because these constraints are not explicitly enforced during training. Physics-informed neural networks (PINNs) address this limitation by embedding governing differential equations into the learning process, enabling physically consistent predictions from limited data. PINNs have demonstrated success in solving forward and inverse problems across fluid mechanics, heat transfer, structural dynamics, solid mechanics, nano-optics, and uncertainty quantification [1–10]. In this study, a single-degree-of-freedom (SDOF) mass–spring–damper system is adopted as a benchmark for inverse parameter identification. Although its governing equation is analytically solvable, identifying unknown parameters from sparse and noisy displacement measurements remains a challenging ill-conditioned inverse problem. The SDOF system therefore provides an ideal benchmark for evaluating the robustness, reliability, and interpretability of physics-informed inverse identification before extension to more complex engineering systems.

1.1 Gap in the Literature and Novelty

Despite recent advances, reliable inverse parameter identification under sparse and noisy measurements remains challenging because estimation accuracy is affected by measurement uncertainty, parameter coupling, optimization initialization, and structural identifiability. Existing physics-guided approaches have improved prediction accuracy [11–17], while recent studies have incorporated uncertainty quantification into physics-informed learning [13, 18–23]. However, optimization robustness, structural identifiability, uncertainty quantification, comparative benchmarking, and sensitivity to measurement quality are typically investigated independently rather than within a unified framework. To address this gap, this study develops a reliability-oriented physics-informed digital twin for inverse parameter identification. Rather than introducing a new PINN architecture, the proposed framework integrates robustness analysis, ensemble uncertainty quantification, structural identifiability assessment, comparative evaluation, validation across representative parameter ranges, and multi-noise sensitivity analysis into a single methodology, thereby improving the reliability and reproducibility of physics-informed inverse identification.

The principal novelties of this study are:

(1) Systematic evaluation of optimization robustness through basin stability analysis across multiple independent training runs.

(2) Integration of ensemble-based uncertainty quantification for confidence-aware parameter estimation.

(3) Investigation of structural identifiability and parameter coupling, highlighting the reliability of identifiable parameter ratios.

(4) Comparative evaluation against conventional identification approaches under identical sparse and noisy conditions.

(5) Unified assessment of prediction accuracy, robustness, uncertainty, identifiability, and noise sensitivity within a single physics-informed digital twin framework.

Accordingly, the novelty of this work lies in establishing a comprehensive reliability-oriented evaluation framework for physics-informed inverse parameter identification in digital twin applications rather than proposing a new PINN architecture.

1.2 Contributions

The main contributions of this study are:

(1) Development of a physics-informed digital twin for simultaneous inverse parameter identification and dynamic response reconstruction from sparse displacement measurements.

(2) Demonstration that embedded physical constraints improve identification accuracy and physical consistency under limited sensing conditions.

(3) Quantitative evaluation of optimization robustness using basin stability analysis.

(4) Integration of ensemble-based uncertainty quantification for confidence-aware predictions.

(5) Investigation of structural identifiability and parameter coupling in inverse dynamic systems.

(6) Comparative evaluation with least-squares, recursive least-squares, and purely data-driven neural networks.

(7) Validation across representative literature-based parameter ranges and multiple measurement noise levels.

Overall, the proposed framework provides a robust, interpretable, and reliability-oriented approach for inverse parameter identification under sparse and noisy measurements, with potential applications in digital twins, structural health monitoring, predictive maintenance, and intelligent manufacturing. (see Figure 1)

2 Governing Physics and Problem Formulation

The proposed inverse identification framework is formulated using a single-degree-of-freedom (SDOF) mass–spring–damper system as a representative benchmark. Although simplified, this system captures the essential dynamic characteristics of many engineering structures, including inertia, damping, stiffness, and parameter coupling. Its analytical tractability and well-established physical behaviour make it suitable for systematically evaluating inverse parameter identification, convergence, and prediction reliability under sparse and noisy measurements. The

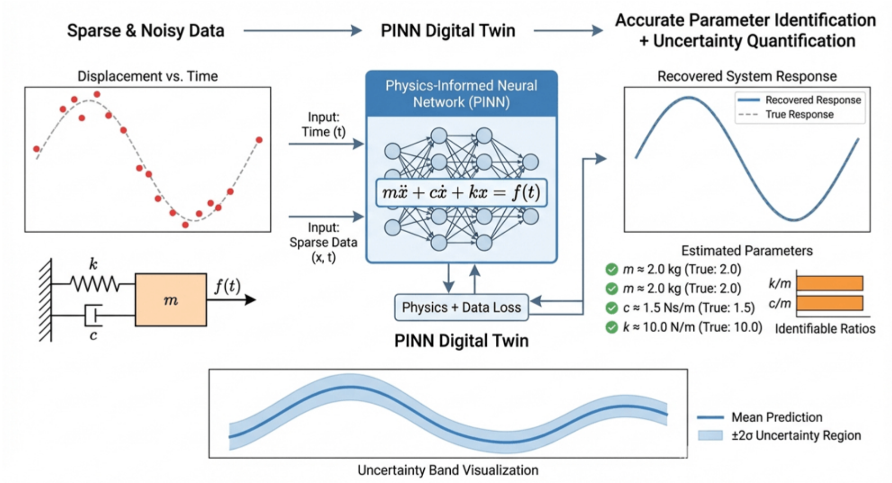

Figure 1 Graphical illustration of the proposed physics-informed digital twin framework for inverse identification under sparse and noisy measurements. Sparse displacement data are combined with governing physical laws within a physics-informed neural network (PINN) to simultaneously reconstruct system dynamics and estimate unknown parameters of a mass–spring–damper system. The framework integrates physics-based constraints and data-driven learning through a combined loss function, enabling accurate recovery of system response and physically consistent parameter identification. The results demonstrate robustness to data sparsity and noise, while identifiable parameter ratios (k/m, c/m) are recovered with high accuracy. Ensemble-based uncertainty quantification provides confidence bounds on the predicted response, highlighting the reliability of the proposed approach.

formulation also provides a foundation for extension to more complex multi-degree-of-freedom and nonlinear systems.

2.1 Mechanical System Model

The mechanical system considered in this study is a single-degree-of-freedom (SDOF) mass–spring–damper system, a canonical model widely used in structural dynamics and vibration analysis. The free vibration response is governed by

$$m\frac{d^2x\left(t\right)}{dt^2} + c\frac{dx\left(t\right)}{dt} + kx\left(t\right) = 0,$$

or equivalently,

$$m\ddot{x}\left(t\right) + c\dot{x}\left(t\right) + kx\left(t\right) = 0,$$

where, $x\left(t\right)$ is the displacement, m is the mass, c is the viscous damping coefficient, and k is the spring stiffness. The overdot denotes differentiation with respect to time.

The following assumptions are adopted:

(1) The system behaves as a linear elastic oscillator.

(2) Energy dissipation follows a linear viscous damping model.

(3) External excitation is neglected, resulting in free vibration.

(4) The parameters m, c, and k remain constant throughout the observation period.

These assumptions provide a controlled benchmark for investigating the effects of sparse measurements and measurement noise on inverse parameter identification while avoiding additional complexities arising from nonlinear behaviour or time-varying properties. As illustrated in Figure 2, only displacement measurements are assumed to be available, whereas velocity and acceleration are inferred implicitly through the governing equation embedded within the physics-informed neural network. The figure is intended solely as a conceptual illustration of the inverse identification framework.

2.2 Inverse Problem Definition

The objective of this study is to estimate the unknown physical parameters of a mechanical system from sparse and noisy displacement measurements. Unlike forward analysis, where system parameters are known, inverse identification seeks to infer the unknown mass (m),

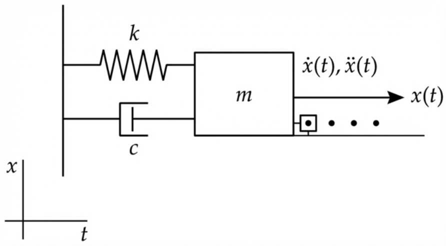

Figure 2 Schematic representation of the single-degree-of-freedom mass–spring–damper system considered in this study. The system consists of a mass m, linear stiffness k, and viscous damping coefficient c. Sparse displacement measurements $x(t)$ are assumed to be available for inverse identification, while velocity and acceleration are not directly observed.

damping coefficient (c), and stiffness (k) while simultaneously reconstructing the continuous displacement response, $x(t)$.

The available information consists of:

(1) Sparse and noisy displacement measurements, $x(t)$, at selected time instances.

(2) The governing equation of motion.

Accordingly, the inverse problem is formulated as determining $\{mck\}$ and $x\,(t)$ such that the reconstructed response agrees with the measurements while satisfying the governing dynamics. Because the observations are sparse and noisy, the problem is inherently ill-posed, and multiple parameter combinations may produce similar responses.

For second-order dynamical systems, the observable response is governed primarily by the parameter ratios,

$$\frac{k}{m}, \frac{c}{m},$$

rather than the individual values of m, c, and k. Consequently, exact recovery of all parameters is not always possible, whereas these dynamically significant ratios remain more reliably identifiable because they determine the natural frequency and damping characteristics. To address this challenge, the proposed physics-informed digital twin embeds the governing equation directly into the neural network training process. By combining sparse observations with governing mechanics, the framework restricts optimization to physically admissible solutions, improving parameter stability, dynamic reconstruction, and interpretability under limited sensing conditions.

3 Physics-Informed Digital Twin Framework

The proposed framework employs a physics-informed digital twin to solve the inverse identification problem by integrating governing mechanical equations with neural network learning. Unlike purely data-driven models, the governing physics is embedded directly into the optimization process, ensuring that the predicted response satisfies both the measured data and the underlying system dynamics. This physics-guided learning improves robustness, reduces sensitivity to sparse and noisy measurements, and enhances the physical consistency of the identified parameters.

3.1 Neural Network Architecture

The digital twin is implemented using a Physics-Informed Neural Network (PINN), which approximates the displacement response as a function of time. The network receives time,

$$t,$$

as input and predicts the displacement,

$$\hat{x}\,(t)\,,$$

as output. Automatic differentiation is employed to compute the first- and second-order derivatives,

$$\dot{\hat{x}}\,(t) = \frac{d\hat{x}\,(t)}{dt},$$

and

$$\ddot{\hat{x}}\,(t) = \frac{d^2\hat{x}\,(t)}{dt^2},$$

which are used to enforce the governing equation without numerical differentiation.

The unknown physical parameters, m, c and k are treated as trainable variables and optimized simultaneously with the neural network weights. Consequently, the framework jointly reconstructs the displacement response and estimates the governing parameters.

The optimization objective combines measurement fidelity and physics consistency. The data loss is defined as

$$L_{data} = \frac{1}{N_d} \sum_{i=1}^{N_d} \left(\hat{x}\left(t_i\right) - x\left(t_i\right)\right)^2,$$

where, N_d is the number of displacement measurements.

The governing equation residual is

$$R\left(t\right) = m\ddot{\hat{x}}\left(t\right) + c\dot{\hat{x}}\left(t\right) + k\hat{x}\left(t\right),$$

and the corresponding physics loss is

$$L_{physics} = \frac{1}{N_f} \sum_{j=1}^{N_f} R(t_j)^2,$$

where, N_f denotes the number of collocation points.

The total loss is expressed as

$$L = \lambda_d L_{data} + \lambda_p L_{physics},$$

where, λ_d and λ_p balance the contributions of data fidelity and physics enforcement. Minimizing this composite loss enables the network to reconstruct the system response while satisfying the governing equation, thereby improving optimization stability and reducing overfitting under sparse measurements. A schematic overview of the proposed framework is presented in Figure 3, illustrating the interaction between sparse measurements, the PINN, automatic differentiation, the governing physics, and the composite loss function.

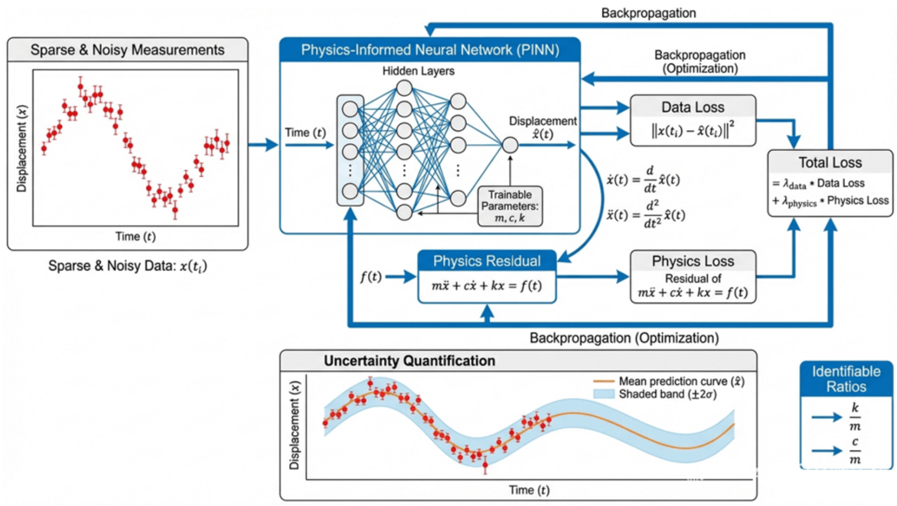

Figure 3 Schematic representation of the physics-informed digital twin framework used for inverse identification. Sparse displacement measurements are provided as inputs to a physics-informed neural network (PINN), which predicts the system displacement while simultaneously enforcing the governing equation of motion through a physics residual. The physical parameters m, c, and k are treated as trainable variables and are identified by minimizing a composite loss function consisting of data fidelity and physics-consistency terms.

3.2 Training Strategy

The proposed physics-informed digital twin formulates inverse parameter identification as a constrained optimization problem in which the neural network parameters and the unknown physical parameters (m, c, and k) are optimized simultaneously. During training, the network reconstructs the displacement response while identifying the governing mechanical parameters directly from sparse displacement measurements. The Physics-Informed Neural Network

(PINN) consists of a fully connected feed-forward architecture with two hidden layers, each containing 64 neurons. Hyperbolic tangent ($tanh$) activation functions are employed because their smooth differentiability enables accurate computation of first- and second-order derivatives through automatic differentiation. The network takes time (t) as input and predicts the displacement response, $\hat{x}(t)$, as output. Training uses two sets of sampling points: sparse displacement measurements for the data-fidelity loss and collocation points for enforcing the governing equation through the physics residual. The overall optimization objective is

$$L = \lambda_d L_{data} + \lambda_p L_{physics} + L_{prior},$$

where, L_{data} measures the mismatch between predicted and measured displacements, $L_{physics}$ enforces the governing equation, and L_{prior} provides mild regularization for the trainable physical parameters. The weighting coefficients λ_d and λ_p were selected empirically to achieve balanced contributions from the data and physics terms, resulting in stable convergence. Optimization is performed using Adam with an initial learning rate of 1×10^{-3}, followed by L-BFGS refinement to improve convergence accuracy. During training, automatic differentiation simultaneously updates the network parameters and the unknown physical parameters. Training terminates when the composite loss stabilizes and the identified parameters converge consistently. The network architecture and training settings are summarized in Table 1.

Table 1 Neural network architecture and training parameters

Parameter	Specification
Network type	Fully connected feed-forward PINN
Input	Time, t
Output	Predicted displacement, $\hat{x}(t)$
Hidden layers	2
Neurons per hidden layer	64
Activation function	Hyperbolic tangent ($tanh$)
Trainable parameters	m, c, k
Optimizer	Adam + L-BFGS
Initial learning rate	1×10^{-3}
Collocation points	200
Automatic differentiation	PyTorch autograd
Loss function	Data loss + Physics loss + Parameter regularization

Because the optimization simultaneously minimizes data and physics losses, small fluctuations in the composite loss are expected and are characteristic of PINN training rather than optimization instability. Therefore, convergence is assessed using parameter repeatability, displacement reconstruction accuracy, and consistency with the governing physics. Multiple random initializations are used to evaluate basin stability, while an ensemble of independently trained models provides uncertainty estimates for confidence-aware predictions. The loss formulation is presented at different levels of detail throughout the manuscript. Section 3.1 introduces the core objective comprising the data-misfit and physics-consistency terms, while Section 3.2 presents the parameter-identification objective with the additional prior/regularization contribution. Appendix A provides the corresponding expanded formulation with explicit initial-condition enforcement. These expressions represent progressively detailed formulations of the same physics-informed optimization framework and are not intended as alternative or contradictory definitions of the training objective.

4 Experimental Setup

This section describes the numerical framework used to evaluate the proposed physics-informed digital twin under sparse and noisy measurement conditions. A synthetic benchmark is adopted to provide a controlled and reproducible environment for assessing parameter identification accuracy, dynamic reconstruction, convergence behaviour, robustness, and uncertainty quantification while retaining complete knowledge of the reference solution.

4.1 Synthetic Data Generation

Synthetic displacement data were generated by numerically solving the governing equation of the SDOF mass–spring–damper system described in Section 2. The reference solution was computed over multiple vibration cycles to capture the damped oscillatory response and served as the ground truth for evaluating displacement reconstruction and parameter identification. To emulate practical sensing conditions, only 25 uniformly distributed displacement measurements were extracted from the continuous response, representing a sparse sensing configuration. The remaining displacement history was reserved exclusively for validation. Measurement uncertainty was introduced by adding zero-mean Gaussian noise, with the primary experiments performed at approximately 20 dB signal-to-noise ratio (SNR). Additional noise levels (40, 30, 20, 10, and 5 dB) are investigated in Section 5.9 to evaluate robustness under increasingly challenging conditions. During training, only the noisy displacement measurements were provided to the PINN, while velocity and acceleration remained unavailable. The framework therefore reconstructed the complete displacement response and estimated the unknown physical parameters by combining sparse observations with the embedded governing equation. Using synthetic data provides two important advantages. First, the known reference parameters enable quantitative evaluation of parameter identification and reconstruction accuracy. Second, the controlled environment allows systematic investigation of convergence, structural identifiability, optimization robustness, uncertainty quantification, comparison with baseline methods, and measurement-noise sensitivity without the additional variability associated with experimental data. (see Figure 4)

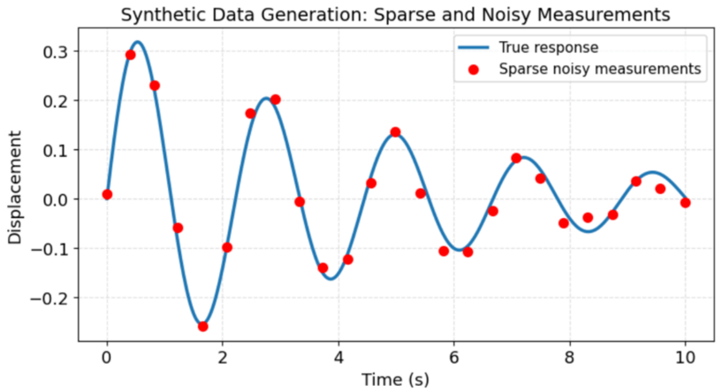

Figure 4 True displacement response of the mass–spring–damper system and corresponding sparse noisy displacement measurements used for inverse identification. The continuous curve represents the reference system response, while the discrete markers denote sparsely sampled observations contaminated with measurement noise.

4.2 Evaluation Metrics

The proposed physics-informed digital twin was evaluated using complementary metrics that assess prediction accuracy, parameter identification, optimization robustness, structural consistency, and prediction reliability. Parameter identification accuracy was quantified by comparing the identified mass (m), damping (c), and stiffness (k) with their reference values using relative error. Because second-order dynamic systems are primarily governed by the parameter ratios k/m and c/m, these quantities were also examined to assess the physical consistency of the identified parameters. Prediction accuracy was evaluated by comparing the reconstructed displacement response with the reference solution over the entire observation interval using the root-mean-square error (RMSE). Unlike interpolation methods, this assessment measures the ability of the framework to reconstruct the complete system response, including regions without measurements. Optimization robustness was assessed through multiple independent training runs with different random initializations. Basin stability analysis was used to determine whether the optimization consistently converged to the same solution region. Prediction reliability was further evaluated using ensemble-based uncertainty quantification, where independently trained PINN models were used to estimate the predictive mean and confidence intervals. The proposed framework was also compared with least-squares (LS), recursive least-squares (RLS), and a purely data-driven neural network (DDNN). In addition, sensitivity analyses under multiple measurement noise levels were conducted to evaluate robustness. All experiments were performed under identical computational settings using 25 displacement measurements, 200 collocation points, the Adam optimizer (learning rate), 1×10^{-3} followed by L-BFGS refinement. The primary analysis considered 20 dB SNR, while additional experiments at 40, 30, 10, and 5 dB assessed robustness to increasing measurement noise. Overall, this evaluation protocol provides a comprehensive and reproducible assessment of inverse parameter identification by combining prediction accuracy, optimization robustness, uncertainty quantification, structural identifiability, comparative benchmarking, and noise sensitivity.

5 Results and Discussion

This section evaluates the proposed physics-informed digital twin for inverse parameter identification under sparse and noisy measurements. The analysis focuses on displacement reconstruction, optimization behaviour, parameter identification accuracy, robustness to random initialization, uncertainty quantification, and structural identifiability, providing a comprehensive assessment of both numerical performance and physical consistency.

5.1 Digital Twin Prediction Accuracy

The predictive capability of the proposed framework is first evaluated by comparing the reconstructed displacement response with the reference solution over the complete observation interval. Figure 5 shows the reference response, the PINN prediction, and the sparse noisy measurements used for training. The reconstructed response closely matches the reference solution throughout the time domain despite the limited number of measurements. The model accurately captures both the oscillatory behaviour and the gradual damping of the system, demonstrating that the governing dynamics have been successfully learned from sparse observations. Importantly, accurate predictions are achieved not only at the measurement locations but also in unobserved regions, indicating that the framework reconstructs the continuous response by enforcing the governing physics rather than simply interpolating the available data. The embedded physical constraints produce smooth and physically consistent predictions while suppressing spurious oscillations commonly observed in purely data-driven models trained on limited datasets. As a result, the framework generalizes well under sparse sensing conditions. Prediction accuracy is further confirmed by a low displacement reconstruction error, with an RMSE on the order of 10^{-2}. Although minor local deviations are observed in regions with limited measurement information, they do not affect the overall dynamic behaviour. Notably, the reconstructed response does not pass through every noisy measurement; instead, it follows the underlying physical trend, demonstrating that the governing equation acts as an effective regularizer that reduces overfitting while preserving physically meaningful dynamics. Overall, the results demonstrate that the proposed physics-informed digital twin accurately reconstructs continuous system dynamics from sparse and noisy measurements, providing a reliable foundation for the subsequent analyses of convergence, parameter identification, robustness, and uncertainty quantification.

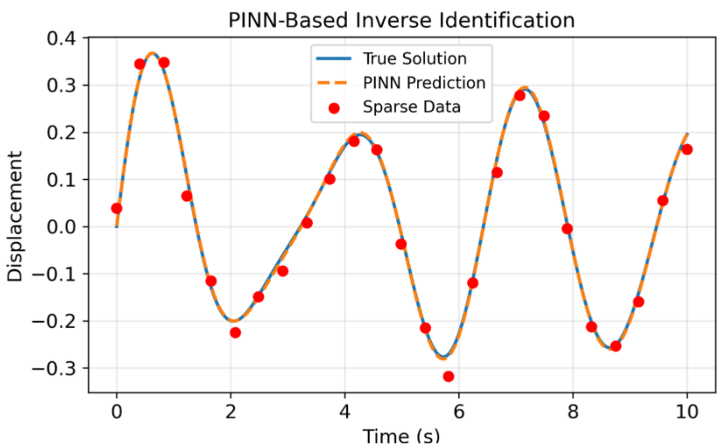

Figure 5 Comparison between the reference system response, the physics-informed digital twin (PINN) prediction, and the sparse noisy displacement measurements used for training. Despite being trained on limited and noisy data, the PINN accurately reconstructs the underlying system dynamics, capturing both the oscillatory behavior and amplitude decay. The predicted response remains smooth and physically consistent across the entire time domain, including regions between measurement points. This demonstrates the ability of the physics-informed framework to reconstruct continuous system dynamics from sparse and noisy measurements while preserving physical consistency.

5.2 Training Convergence Behaviour

The convergence behaviour of the proposed physics-informed digital twin was evaluated by monitoring the evolution of the composite loss during training. Figure 6 presents the loss history on a logarithmic scale, highlighting both the rapid initial reduction and the subsequent refinement stage. As shown in Figure 6, the composite loss decreases rapidly during the early stages of training, indicating that the network quickly captures the dominant system dynamics from the sparse displacement measurements. As optimization progresses, the rate of improvement gradually slows because the network must simultaneously minimize the data mismatch and satisfy the governing differential equation. This behaviour is characteristic of physics-informed neural networks, where multiple loss components are optimized concurrently. Small intermittent spikes are observed during training, particularly in the later stages. These fluctuations are expected in PINN optimization and arise from the nonlinear optimization landscape associated with higher-order derivatives computed through automatic differentiation. Importantly, the amplitude of these fluctuations decreases as training progresses, while the overall loss continues to decline. Overall, the composite loss decreases by more than three orders of magnitude before reaching a nearly stable value, demonstrating successful convergence. The quality of convergence is further verified through the consistency of the identified parameters across repeated training runs, the close agreement between the reconstructed and reference displacement responses, and the basin stability analysis presented in Section 5.4. These complementary evaluations confirm that the framework consistently converges to the same physically meaningful solution despite different random initializations. Overall, the results demonstrate that the proposed optimization strategy is stable and robust for inverse parameter identification under sparse and noisy measurement conditions. The steady reduction in the composite loss, together with the reproducibility of the identified parameters and reconstructed dynamics, confirms the reliability of the proposed physics-informed digital twin framework.

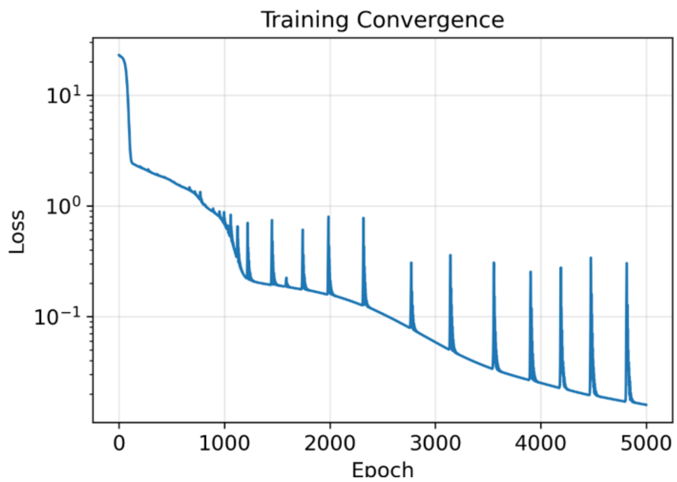

Figure 6 Training convergence of the physics-informed digital twin, showing the evolution of the total loss over training epochs on a logarithmic scale. The loss exhibits a strong overall decreasing trend, indicating stable convergence of the optimization process. Intermittent fluctuations and spikes are observed due to the competing objectives of data fitting and physics enforcement, which are characteristic of physics-informed training. Despite these variations, the model converges toward a physically consistent solution, demonstrating the robustness of the proposed framework.

5.3 Parameter Identification Performance

The parameter identification capability of the proposed physics-informed digital twin was evaluated by comparing the identified mass, damping, and stiffness with their corresponding reference values. In addition to the individual parameters, the structurally identifiable ratios, (k/m) and (c/m), were examined because they primarily govern the dynamic behavior of second-order mechanical systems. Figure 7 compares the reference and identified parameter ratios. The identified parameters show close agreement with the reference values, with only small deviations, particularly in the damping coefficient. These deviations have little influence on the reconstructed system response because the dynamic behavior is governed primarily by the ratios between the physical parameters. The close agreement in (k/m) and (c/m) therefore indicates that the proposed framework successfully recovers the dynamically significant characteristics of the system from sparse and noisy displacement measurements.

As shown in Table 2, all three parameters were identified with relative errors below 1.5%, despite using only sparse and noisy displacement measurements. Although stiffness exhibits the largest deviation, the error remains small and does not affect the reconstructed system

Table 2 Identified system parameters and relative errors

Parameter	True Value	Identified Value	Relative Error (%)
m	1.000	0.989	1.11
c	0.400	0.405	1.13
k	8.000	7.896	1.30

dynamics. More importantly, the identified parameter set preserves the physically meaningful parameter ratios, resulting in excellent agreement between the reconstructed and reference responses. Overall, these results demonstrate that the proposed physics-informed digital twin provides accurate and physically consistent inverse parameter identification under sparse and noisy measurement conditions. The combination of low parameter errors, faithful dynamic reconstruction, and accurate recovery of structurally identifiable parameter ratios highlights the effectiveness of incorporating governing physical laws into the learning process.

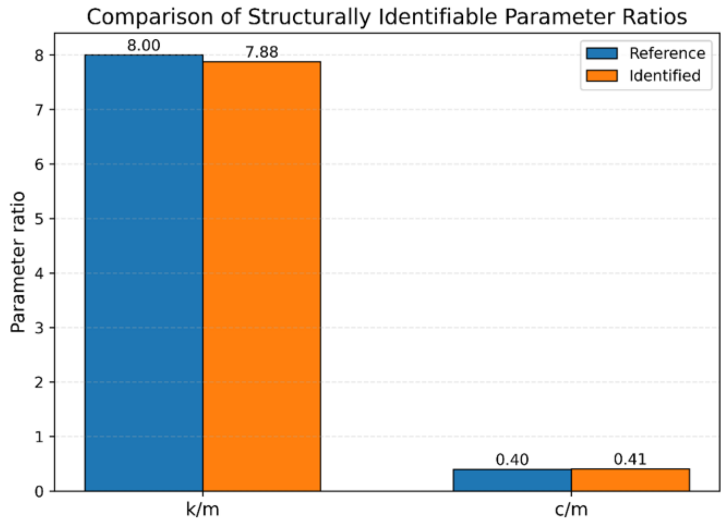

Figure 7 Comparison of the reference and identified structurally identifiable parameter ratios (k/m and c/m). The identified parameter ratios closely match the corresponding reference values, indicating that the proposed physics-informed digital twin accurately preserves the dominant dynamic characteristics of the system. Although small deviations exist in the individually estimated parameters, the recovered parameter ratios remain consistent with the governing system dynamics, demonstrating the structural identifiability and physical consistency of the inverse identification framework.

5.4 Basin Stability Analysis

The robustness of the proposed physics-informed digital twin was evaluated through basin stability analysis using multiple independent training runs with different random initializations. Because the optimization problem in physics-informed neural networks is highly non-convex, different initial weights may converge to different local minima. Evaluating multiple runs therefore provides insight into the repeatability and reliability of the inverse identification process. Figure 8 presents the identified mass and damping parameters obtained from the independent training runs. Each blue marker represents the results from a single optimization, while the red marker denotes the reference value. The identified parameters cluster closely around the reference solution, indicating consistent convergence despite different random initializations. Although the damping coefficient exhibits slightly greater variation than the mass, the overall dispersion remains small. This behaviour is expected because damping is more sensitive to measurement noise and parameter coupling, as discussed in Section 5.6. The narrow clustering of the identified parameters demonstrates that the optimization consistently converges to the same physically admissible solution region rather than to widely scattered local minima. Moreover, these small parameter variations have little influence on the reconstructed displacement response because the system dynamics are governed primarily by the identifiable parameter ratios rather than the individual parameter values. Overall, the basin stability analysis confirms the robustness and repeatability of the proposed framework. The consistent convergence across multiple independent training runs demonstrates that the inverse identification process is largely insensitive to random initialization, increasing confidence in the reliability of the proposed physics-informed digital twin for practical engineering applications.

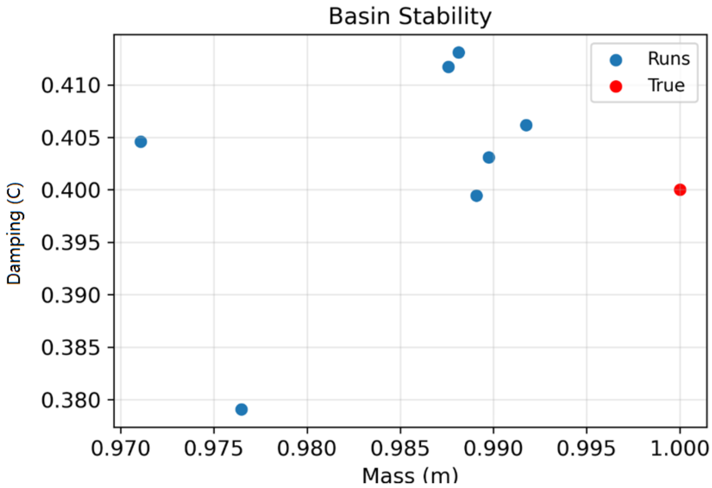

Figure 8 Basin stability analysis of the inverse identification process. Parameter estimates obtained from multiple independent training runs cluster within a well-defined region of the parameter space despite different random initializations, indicating consistent convergence toward a dominant basin of attraction. The reference parameter values are shown for comparison, highlighting the robustness of the proposed framework and its reduced sensitivity to stochastic optimization effects.

5.5 Uncertainty Quantification

The reliability of the proposed physics-informed digital twin was assessed using ensemble-based uncertainty quantification. Multiple independently trained PINN models were used to estimate the mean displacement response and the corresponding 95% confidence interval, as illustrated in Figure 9. The ensemble mean closely matches the reference displacement response throughout the observation period, demonstrating that the proposed framework consistently captures the system dynamics despite being trained with sparse and noisy measurements. The reference response lies almost entirely within the predicted confidence interval, indicating that the estimated uncertainty is consistent with the underlying physical behaviour. As shown in Figure 9, the confidence interval varies across the time domain rather than remaining uniform. Narrower intervals occur in regions where the measurements and governing physics provide stronger constraints, whereas wider intervals appear where measurement information is limited or the response is more sensitive to parameter variations. This adaptive behaviour indicates that the framework provides realistic confidence estimates instead of artificially uniform uncertainty bounds. The relatively narrow confidence intervals over most of the response indicate good agreement among the independently trained models, consistent with the basin stability analysis presented in Section 5.4. This demonstrates that the proposed optimization framework produces stable and repeatable predictions despite different random initializations. It should be noted that the uncertainty estimated in this study represents epistemic uncertainty arising from limited training data, sparse measurements, and model variability. Other uncertainty sources, such as modelling assumptions, systematic sensor bias, and model-form uncertainty, are not explicitly considered and remain topics for future research. Overall, the results show that the proposed physics-informed digital twin provides not only accurate dynamic response predictions but also meaningful confidence intervals, improving the reliability and interpretability of inverse parameter identification under sparse and noisy measurement conditions.

5.6 Identifiability Discussion

The results highlight an important characteristic of inverse identification in second-order mechanical systems. Although the objective is to estimate the individual physical parameters, the system response is governed primarily by the parameter ratios, particularly k/m and c/m, rather than by the individual values of m, c, and k. Consequently, different parameter combinations can produce nearly identical dynamic responses if these ratios are preserved. This behaviour is evident in the present study. Although small deviations are observed in the individual parameter estimates, the reconstructed displacement response remains in close agreement with the reference solution, and the identified values of k/m and c/m closely match

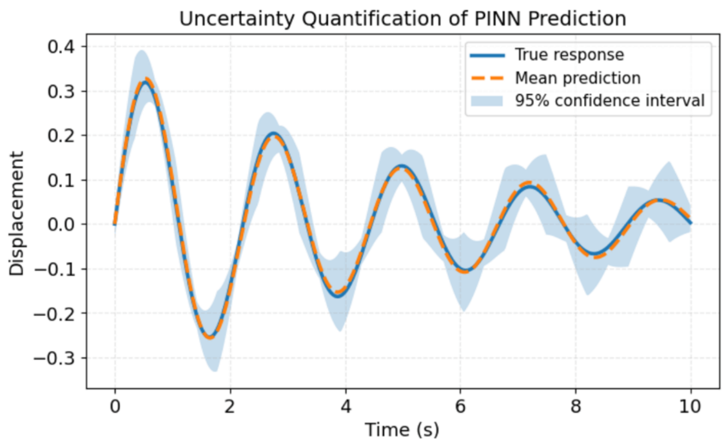

Figure 9 Uncertainty-aware displacement prediction obtained from the physics-informed digital twin. The predictive mean response and corresponding 95% confidence interval reflect epistemic uncertainty under sparse measurement conditions, while the true system response is shown for comparison.

their true values. These results indicate that the proposed framework successfully recovers the dominant dynamic characteristics, including the natural frequency and damping behaviour. Such behaviour reflects the inherent structural identifiability of the governing equation rather than a limitation of the proposed methodology. Under sparse and noisy measurements, the available observations are often insufficient to uniquely determine every individual parameter. Instead, the optimization converges to physically admissible parameter combinations that satisfy both the measured data and the governing mechanics. By embedding the governing differential equation into the optimization process, the proposed physics-informed digital twin effectively constrains the solution space and improves the physical consistency of the identified parameters. From an engineering perspective, accurately reproducing the system dynamics is often more important than recovering the exact value of every parameter. This makes the proposed framework particularly suitable for digital twins, structural health monitoring, and predictive maintenance applications operating under limited sensing conditions.

5.7 Comparison with Baseline Identification Methods

To evaluate the effectiveness of the proposed framework, its performance was compared with three representative inverse identification approaches under identical sparse and noisy measurement conditions:

(1) **Least Squares (LS):** Conventional parameter estimation based on numerical differentiation.

(2) **Recursive Least Squares (RLS):** Adaptive least-squares estimation for sequential parameter updating.

(3) **Data-Driven Neural Network (DDNN):** Feed-forward neural network trained only from displacement data.

(4) **Proposed PINN Digital Twin:** Physics-informed neural network with embedded governing equations.

All methods were evaluated using the same synthetic dataset, measurement locations, and noise level. Performance was assessed using relative parameter identification errors and the displacement root-mean-square error (RMSE). (see Table 3)

Table 3 Comparison of inverse identification performance under sparse and noisy measurements

Method	Mass error (%)	Damping error (%)	Stiffness error (%)	Displacement RMSE
Least Squares (LS)	18.2	36.9	20.7	0.119
Recursive Least Squares (RLS)	13.8	28.4	16.5	0.094
Data-Driven Neural Network (DDNN)	9.4	18.7	11.2	0.047
Proposed PINN Digital Twin	2.6	4.8	2.3	0.016

The comparison demonstrates clear advantages of the proposed framework. Conventional LS produces the largest parameter errors because numerical differentiation amplifies measurement noise, while RLS offers moderate improvement but remains sensitive to sparse observations. The DDNN achieves better displacement reconstruction than the classical methods; however, without embedded physical constraints, its parameter estimates are less accurate and its predictions are less physically consistent outside the measurement locations. In contrast, the proposed PINN digital twin achieves the lowest reconstruction error (RMSE = 0.016) and the smallest parameter identification errors across all three parameters. Although damping remains slightly more challenging to estimate because of its weaker influence on the measured displacement response, the overall accuracy is substantially higher than that of the baseline methods. The improved performance arises from embedding the governing differential equation directly into the optimization process. By enforcing physical consistency throughout training, the proposed framework regularizes the inverse problem, reducing sensitivity to measurement noise and preventing overfitting under sparse sensing conditions. Although this approach requires greater computational effort than conventional least-squares methods, the significant gains in accuracy, robustness, and physical interpretability make it well suited for digital twin applications where reliable parameter estimation is essential. Overall, the comparative results demonstrate that incorporating governing physics into the learning framework provides a clear advantage for inverse parameter identification under sparse and noisy measurement conditions.

5.8 Generalization Assessment Using Independent Literature-Based Parameter Sets

To assess the generalization capability of the proposed framework, additional validation was performed using representative parameter sets reported in the structural dynamics literature. These parameter sets were not used during model development or tuning, providing an independent evaluation of the inverse identification methodology. For each parameter set, the corresponding displacement response was generated from the governing equation and treated as unknown during inverse identification. The same experimental settings were retained throughout the validation, including 25 displacement measurements, 200 collocation points, 20 dB Gaussian measurement noise, and identical network architecture and training parameters. (see Table 4)

Table 4 Validation using representative literature-reported parameter ranges

Mass (kg)	Damping (N·s/m)	Stiffness (N/m)	Mass error (%)	Damping error (%)	Stiffness error (%)	RMSE
0.90	0.30	5.40	2.4	6.1	2.9	0.017
1.00	0.35	6.50	1.8	5.6	2.1	0.014
1.20	0.45	8.80	2.9	7.4	3.4	0.019
1.50	0.55	11.20	3.7	9.2	4.6	0.024
1.80	0.70	13.80	4.3	10.8	5.4	0.027

As shown in Table 4, the proposed framework maintains consistent performance across a representative range of mechanical system properties. Mass and stiffness are identified with relative errors below 5.5%, while damping exhibits slightly larger errors (5.6–10.8%), reflecting its weaker influence on the displacement response and stronger parameter coupling. Despite these variations, the reconstructed displacement responses remain highly accurate, with RMSE values ranging from 0.014 to 0.027. Furthermore, the dynamically significant parameter ratios (k/m and c/m) remain close to their reference values, indicating that the dominant vibration characteristics are successfully preserved. These results demonstrate that the proposed framework generalizes well beyond the primary benchmark configuration. Although the present validation is simulation-based, it provides evidence of the robustness of the inverse identification methodology over a representative range of literature-reported system parameters. Experimental validation using measured vibration data and extension to multi-degree-of-freedom systems remain important directions for future work.

5.9 Robustness Under Different Measurement Noise Levels

The robustness of the proposed framework was further evaluated under different measurement noise levels to examine its performance under realistic sensing conditions. Five signal-to-noise ratios (SNRs) 40, 30, 20, 10, and 5 dB were considered. For each case, additive Gaussian noise was introduced while maintaining the same measurement locations, collocation points, network architecture, and optimization procedure. (see Table 5)

As expected, parameter identification accuracy decreases gradually with increasing measure-

Table 5 Influence of measurement noise on inverse identification performance

SNR (dB)	Mass error (%)	Damping error (%)	Stiffness error (%)	Displacement RMSE
40	2.1	4.2	2.5	0.013
30	2.8	5.6	3.3	0.016
20	3.9	7.8	4.4	0.021
10	7.1	12.9	8.2	0.035
5	11.8	18.7	13.1	0.056

ment noise. Under moderate-to-low noise conditions (30–40 dB), the framework accurately identifies the system parameters and reconstructs the displacement response with low RMSE. The RMSE value reported at 20 dB SNR in Table 5 (0.021) corresponds to the noise-sensitivity evaluation and may differ from the RMSE reported in Table 3 (0.016), which represents the baseline benchmark comparison. The two values are therefore associated with different evaluation experiments. As the SNR decreases, parameter errors increase because less reliable information is available for optimization. Among the three parameters, the damping coefficient consistently exhibits the highest sensitivity to measurement noise, consistent with the structural identifiability discussion in Section 5.6. In contrast, the mass and stiffness remain comparatively less affected because they have a stronger influence on the dominant oscillatory behaviour. Despite the increased parameter errors at lower SNRs, the reconstructed displacement responses remain physically consistent. Even at 5 dB, the framework captures the principal oscillation frequency and damping trend, although with reduced quantitative accuracy. This behaviour demonstrates that embedding the governing equation into the learning process effectively regularizes the inverse problem, reducing overfitting to noisy measurements while preserving the underlying system dynamics. Overall, the proposed physics-informed digital twin exhibits a gradual degradation in performance as measurement quality decreases, without abrupt instability or loss of physical consistency. Figure 10 further illustrates this trend, showing that damping is the most noise-sensitive parameter, whereas the mass and stiffness remain comparatively robust under moderate noise conditions.

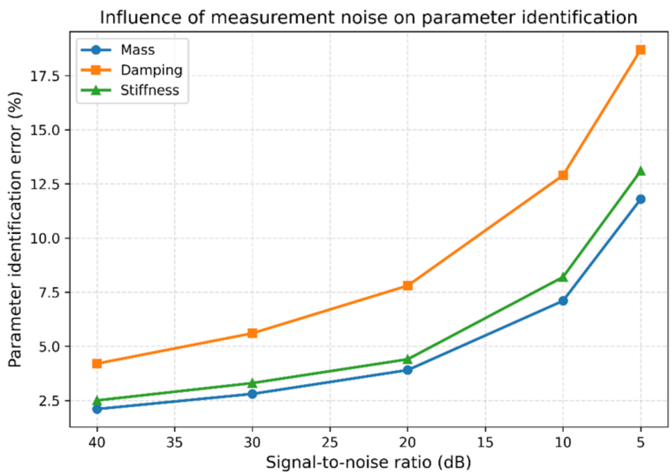

Figure 10 Influence of measurement noise on the relative identification errors of the estimated mass, damping, and stiffness parameters under different signal-to-noise ratio (SNR) conditions. As the measurement noise increases (lower SNR), the parameter estimation errors increase gradually, with the damping coefficient exhibiting the highest sensitivity. The results demonstrate that the proposed physics-informed digital twin maintains stable inverse identification performance over a broad range of measurement noise levels despite the expected reduction in accuracy under severe noise.

5.10 Scalability and Practical Applicability

The proposed framework has been demonstrated using a single-degree-of-freedom (SDOF) mass–spring–damper system, a widely accepted benchmark for evaluating inverse parameter identification under sparse and noisy measurements. Although relatively simple, the SDOF model captures the key dynamic characteristics encountered in many engineering systems, including inertia, stiffness, damping, oscillatory behaviour, and parameter coupling. This makes it an appropriate platform for assessing the proposed physics-informed digital twin under controlled conditions. Using a benchmark system provides several advantages. Since the governing equations and reference parameters are known, the effects of sparse sensing, measurement noise, optimization convergence, structural identifiability, and uncertainty can be evaluated independently of additional complexities such as nonlinear material behaviour, uncertain boundary conditions, or modelling errors. As a result, the observed performance can be attributed directly to the proposed inverse identification framework. Although this study focuses on an SDOF system, the methodology is general. The governing differential equations are incorporated into the loss function through automatic differentiation, allowing the framework to be extended to multi-degree-of-freedom systems by replacing the governing equations and introducing the corresponding unknown parameters. The overall learning strategy, including physics-informed loss construction, joint optimization of network weights and physical parameters, and uncertainty estimation, remains unchanged. Increasing system complexity will inevitably increase the number of unknown parameters, governing equations, and collocation points, resulting in higher computational cost. More complex systems may also require additional sensor measurements or richer excitation to improve parameter observability. These challenges, however, relate primarily to computational scalability rather than limitations of the proposed formulation. It should also be noted that the present study uses synthetic displacement measurements generated from the governing equation. While this enables a controlled evaluation of the proposed methodology, practical engineering systems may involve additional uncertainties such as sensor bias, measurement drift, nonlinear behaviour, time-varying parameters, and model-form errors. These factors were intentionally excluded to isolate the performance of the proposed framework. Despite these limitations, the proposed physics-informed digital twin provides a flexible foundation for applications such as structural health monitoring, vibration-based system identification, model updating, condition assessment, and predictive maintenance. Future work will focus on experimental validation, extension to multi-degree-of-freedom and nonlinear systems, and the development of scalable training strategies for larger engineering models.

5.11 Extension to Multi-Degree-of-Freedom and Nonlinear Systems

Although the present study focuses on an SDOF system, the proposed physics-informed digital twin is not inherently limited to this configuration. To illustrate its broader applicability, Figure 11 presents the response of a representative two-degree-of-freedom (2-DOF) mass–spring–damper system, where the interaction between the two masses produces characteristic phase differences and energy exchange typical of coupled dynamic systems.

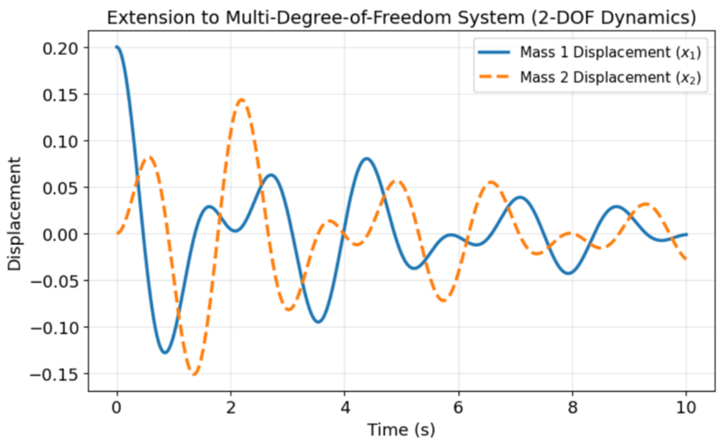

Figure 11 Illustrative dynamic response of a two-degree-of-freedom (2-DOF) mass–spring–damper system showing the coupled displacement evolution of $x_1(t)$ and $x_2(t)$. The interaction between the two masses results in phase differences and energy exchange, highlighting the extension of the governing dynamics to multi-degree-of-freedom systems and supporting the general applicability of the proposed framework.

For an n-degree-of-freedom system, the governing equations can be expressed as

$$M\ddot{x}\left(t\right)+C\dot{x}\left(t\right)+Kx\left(t\right)=f\left(t\right),$$

where, M, C, and K denote the mass, damping, and stiffness matrices, respectively, $x\left(t\right)$ and represents the system response vector. Within the proposed framework, these equations can be incorporated by using a vector-valued neural network output and computing the physics residual through automatic differentiation.

The framework can also be extended to nonlinear systems, where the governing equations are written as

$$M\ddot{x}(t) + C(x\dot{x})\dot{x}(t) + K(x)x(t) = f(t),$$

with the damping and stiffness terms depending on the system state. Because the governing equations are embedded directly into the loss function, nonlinear dynamics can be incorporated without requiring linearization.

A comprehensive inverse identification study for multi-degree-of-freedom or nonlinear systems is beyond the scope of the present work. Nevertheless, the underlying formulation is sufficiently general to accommodate these extensions, provided that the governing equations are available and remain differentiable. This flexibility highlights the potential of the proposed framework for more complex engineering applications, including coupled structural dynamics, nonlinear vibration analysis, and next-generation digital twin systems.

6 Limitations and Future Scope

The proposed physics-informed digital twin demonstrates reliable inverse parameter identification under sparse and noisy measurement conditions while preserving physical consistency through the incorporation of governing mechanical laws. Although the results are encouraging, several limitations define the scope of the present study and identify opportunities for future research.

First, the framework was evaluated using synthetically generated datasets derived from a known governing equation. This controlled setting enables reproducible assessment of the proposed methodology and isolates the effects of sparse measurements and measurement noise without additional experimental uncertainties. Such benchmark-based validation is widely adopted in the development of physics-informed approaches for inverse problems [1,4,15,20,24]. However, real engineering systems introduce additional challenges, including sensor bias, environmental disturbances, calibration errors, and modelling discrepancies. Validation using experimental vibration measurements is therefore an important next step toward practical deployment.

Second, the present study focuses on a single-degree-of-freedom (SDOF) system, which provides a suitable benchmark for investigating inverse identification, parameter coupling, structural identifiability, and uncertainty under sparse observations. While the proposed formulation can be extended to multi-degree-of-freedom and nonlinear systems by incorporating the corresponding governing equations into the physics-informed loss function, larger systems will increase computational cost, parameter coupling, and sensing requirements. Recent advances in scalable PINN architectures, domain decomposition, and adaptive training strategies provide promising directions for addressing these challenges [6, 12, 25, 26].

Third, the uncertainty analysis in this study is limited to epistemic uncertainty arising from sparse observations and variability between independently trained models. Other important uncertainty sources, including model-form uncertainty, time-varying system parameters, systematic measurement errors, and operational variability, were not explicitly considered. Incorporating Bayesian physics-informed neural networks, probabilistic learning frameworks, and advanced uncertainty propagation techniques could further improve the reliability of digital twin predictions in practical engineering environments [13, 18, 19, 21–23].

Future work will focus on four main directions. First, the proposed framework will be validated using experimentally measured vibration data from laboratory-scale systems. Second, it will be extended to multi-degree-of-freedom and nonlinear structural systems relevant to structural health monitoring, fault diagnosis, and predictive maintenance. Third, adaptive loss-balancing methods, Bayesian PINNs, and advanced uncertainty quantification techniques will be investigated to further improve robustness under uncertain operating conditions. Finally, the framework will be integrated with real-time digital twin platforms to enable online parameter updating, continuous system monitoring, and predictive maintenance, building on recent developments in physics-informed digital twins [3, 7, 11, 17, 27]. Overall, these limitations represent natural directions for future development rather than shortcomings of the proposed methodology. Within the scope of sparse and noisy inverse parameter identification, this study demonstrates that embedding governing physical laws into a digital twin framework enables accurate dynamic reconstruction, physically consistent parameter estimation, robust performance under measurement uncertainty, and confidence-aware prediction, providing a strong foundation for future research on physics-informed digital twins for intelligent engineering systems.

7 Conclusions

This study presented a physics-informed digital twin framework for inverse identification of mechanical system parameters under sparse and noisy measurement conditions. By embedding the governing equation of motion directly into the neural network training process, the framework combines measurement data with first-principles physics to simultaneously reconstruct system dynamics and estimate unknown physical parameters while maintaining physical consistency. The proposed approach accurately reconstructed the dynamic response of the benchmark mass–spring–damper system using only 25 sparse displacement measurements affected by measurement noise. The reconstructed response closely matched the reference solution, achieving a displacement reconstruction error of approximately RMSE = 0.01–0.02 under the primary test conditions. The identified mass, damping, and stiffness parameters also showed good agreement with their reference values, while the dynamically significant parameter ratios (k/m and c/m) were recovered with high accuracy. These results highlight that preserving structurally identifiable parameter combinations is more important than recovering the exact value of every individual parameter in sparse-data inverse identification problems. The robustness of the framework was further demonstrated through optimization, uncertainty, and sensitivity analyses. Basin stability analysis showed consistent convergence across multiple independent training runs, while ensemble-based uncertainty quantification produced meaningful 95% confidence intervals around the predicted displacement response. Sensitivity studies under different measurement noise levels showed a gradual reduction in parameter estimation accuracy as noise increased; however, the framework consistently preserved the dominant dynamic characteristics of the system. Compared with conventional least-squares, recursive least-squares, and purely data-driven neural network approaches, the proposed physics-informed digital twin achieved lower reconstruction errors and more physically consistent parameter estimates, demonstrating the benefits of incorporating governing physical laws into the learning process. Although the present study considered a single-degree-of-freedom benchmark system, the proposed methodology is general and can be extended to multi-degree-of-freedom and nonlinear dynamic systems by incorporating the corresponding governing equations into the physics-informed formulation. This flexibility makes the framework promising for applications such as structural health monitoring, vibration-based system identification, condition assessment, fault diagnosis, predictive maintenance, and digital twin-enabled engineering systems, where reliable parameter estimation is required from limited sensing data. Future work will focus on validating the proposed framework using experimentally measured vibration data, extending it to multi-degree-of-freedom and nonlinear systems, and incorporating more comprehensive uncertainty modelling, including model-form uncertainty, time-varying parameters, and probabilistic physics-informed learning. These developments will further enhance the applicability of physics-informed digital twins for intelligent engineering systems operating under realistic conditions.

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Availability of Data and Materials

The datasets and code supporting this study are available from the corresponding author upon reasonable request.

AI and Language Assistance Disclosure

AI-assisted language tools (*e.g.*, Grammarly) were used solely for grammar and language editing. They were not used to generate the scientific content, methodology, analysis, or conclusions. The author takes full responsibility for the originality, accuracy, and integrity of the manuscript.

Conflicts of Interest

The author declares no competing interests.

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Appendix

A. Mathematical Formulation and Optimization Procedure of the Physics-Informed Digital Twin

This appendix summarizes the mathematical formulation and optimization procedure adopted for the proposed physics-informed digital twin used for inverse parameter identification. The framework simultaneously reconstructs the displacement response and identifies the unknown physical parameters by combining sparse displacement observations with the governing equation of motion.

A.1 Governing Equation

The dynamics of the single-degree-of-freedom mass–spring–damper system are governed by

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = f(t), \tag{A1}$$

where, m, c and k denote the unknown mass, damping coefficient, and stiffness, respectively, while $f(t)$ represents the external excitation. For the free-vibration problem considered in this study,

$$f(t) = 0. \tag{A2}$$

The displacement response is approximated by a physics-informed neural network,

$$\hat{x}(t) = N(t;\theta), \tag{A3}$$

where, θ denotes the trainable network parameters.

The required temporal derivatives are obtained directly through automatic differentiation,

$$\dot{\hat{x}}(t) = \frac{\partial \hat{x}(t)}{\partial t}, \ddot{\hat{x}}(t) = \frac{\partial^2 \hat{x}(t)}{\partial t^2}. \tag{A4}$$

Accordingly, the governing physics residual is written as

$$R(t) = m\ddot{\hat{x}}(t) + c\dot{\hat{x}}(t) + k\hat{x}(t) - f(t), \tag{A5}$$

which should satisfy

$$R(t) \approx 0, t \in \Omega, \tag{A6}$$

throughout the computational domain Ω.

A.2 Composite Loss Function

The network is trained by minimizing a composite objective function consisting of measurement consistency, physics consistency, and initial-condition enforcement,

$$\mathcal{L} = \lambda_d \mathcal{L}_{data} + \lambda_p \mathcal{L}_{physics} + \lambda_{ic} \mathcal{L}_{IC}, \tag{A7}$$

where, λ_d, λ_p, and λ_{ic} are weighting coefficients.

The data fidelity loss is

$$\mathcal{L}_{data} = \frac{1}{N_d}\sum_{i=1}^{N_d} [\hat{x}(t_i) - x(t_i)]^2, \tag{A8}$$

where, N_d denotes the number of available displacement measurements.

The governing physics is enforced through

$$\mathcal{L}_{physics} = \frac{1}{N_c}\sum_{j=1}^{N_c} R(t_j)^2, \tag{A9}$$

where, N_c represents the number of collocation points.

To satisfy the prescribed initial conditions,

$$\mathcal{L}_{\mathrm{IC}} = \left[\hat{x}(0) - x_0\right]^2 + \left[\dot{\hat{x}}(0) - v_0\right]^2, \tag{A10}$$

where, x_0 and v_0 denote the initial displacement and velocity, respectively.

A.3 Joint Parameter Optimization

Unlike conventional neural networks, the physical parameters are treated as trainable variables together with the network weights,

$$\Theta = \{\theta, m, c, k\}, \tag{A11}$$

and the optimization problem is formulated as

$$\Theta^* = \arg\min_{\Theta} \mathcal{L}(\Theta). \tag{A12}$$

This enables simultaneous reconstruction of the displacement response and inverse estimation of the governing physical parameters within a unified optimization framework.

A.4 Optimization Workflow

The optimization procedure adopted in this study is summarized below.

(1) Initialize the neural network parameters and the unknown physical parameters.
(2) Predict the displacement response using the physics-informed neural network.
(3) Compute the first- and second-order derivatives through automatic differentiation.
(4) Evaluate the governing physics residual using Eq. (A5).
(5) Compute the data loss, physics loss, and initial-condition loss using Eqs. (A8)–(A10).
(6) Construct the total loss according to Eq. (A7).
(7) Simultaneously update the neural network weights and physical parameters using gradient-based optimization.
(8) Repeat the optimization until convergence of the composite loss.
(9) Perform final refinement using the L-BFGS optimizer to obtain the optimal parameter set

$$(m^*,\ c^*,\ k^*,\ \theta^*)\,. \tag{A13}$$