RESEARCH ARTICLE

Living systems allometric scaling laws

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Abstract: The spatial and temporal structuring and functioning of living systems are associated with scaling independent qualitative characteristics (gauge invariance) and quantitative laws (power laws). This is allowed by the emergence of new blueprints through the systems merging into ‘Associations for the Reciprocal and Mutual Sharing of Advantages and Dis-Advantages’ (ARMSADA). The local actors become more and more mutually integrated into their new global Whole. Then they are more and more independent from their previous local situations of emergence. Reversely (systemic constructal law), the global Whole is more and more integrating local parceners. The relationship between actors within a living system was described using allometric laws, e.g. the metabolic rate of a lot of species was supposed to be proportional to its mass according to a 3/4 exponent power-law (Kleiber’s law). But, according to the gauge invariance paradigm, an other explanation of the invariant scaling of living systems is proposed with a 2/3 power-law. Whatever its level of organization, a living system, ‘system of systems’ emerging by embedments and juxtapositions of previous ones, effectively functions in 4 dimensions (VA: the Adult system Volume, and tg: the time of generation, the duration that is necessary to acquire the capacity of reproduction). Looking at the gauge invariance paradigm as a ‘factual’ system, from the quantum of Planck to the Universe as a Whole, a meta-analysis of a database of the systems internal (endophysiotope) and external (ecoexotope) interactions can allow to quantify 45×18 allometric relationships. This allows to evidence a ‘grammar’: 1. Invariant independent processes (power-laws with exponent $\varepsilon=0$); 2. Simultaneous limiting interactions regulation processes ($\varepsilon=+1$); 3. Feedback ($\varepsilon=-1$); 4. Competition between actors ($\varepsilon=1/2$); 5. Optimal exchanges flow ($\varepsilon=2/3$) processes. Brownian motion is the basic fundamental process that governs all functions. From the Monera to the ecosystem levels the increasing of regulation processes allows more and more autonomy of the endophysiotope from the ecoexotope dependence. From the point of view of matter and energy flows, living systems optimize the input and output exchanges at their interface. The greater diversity of regulation processes occurs for the endophysiotope throughput flows. Whatever the organization level, living systems optimize their survival by adjusting ‘the capacity to be hosted’ of their endophysiotope (HOSTED) to the changes of ‘the hosting capacity’ of their ecoexotope (HOSTING).

Keywords: living systems, allometric scaling, systemic constructal law, Power-Laws, Pieron’s law

1 Introduction

A swarm of bees is not a population of organisms, but an organism which regulates its internal temperature depending on the external one.[1] Into the Whole (the swarm system), the actors (the bees) are in interaction, as in our organism (the Whole), our cells (the actors) are, they are functionally defined by their endophysiotope (ENDO: internal, tope: space, physio: of functioning) and their ecoexotope (EXO: external, tope: space, eco: of inhabitation), that together define the system as a Wholeness,[1] and the interface of exchange between ENDO and EXO (Figure 1). Every living system is hosted by a hosting environment,[3] integrated into food chains, it is a guest of an ecoexotope (EXO), in which it is a parcener,[1] as every other living forms.[3] In order to survive it must first ‘to eat’. Then, if it may ‘not to be eaten’, if its endophysiotope (ENDO) can grow in mass, it must itself ‘to survive its self’, to generate an offspring and eventually to grow in number. Food chains are juxtaposed and encased (Figure 1) like times and spaces are.[5,6] Every living system, like a cellular organism (at the microscopic level), or a multi-meta-cellular organism (at the macroscopic one) or an ecosystem, which is a multi-meta-((multimeta)-cellular) organism, is entirely defined and governed by 7 functional characteristics (Figure 2) that are mutually necessary and sufficient for its duration and that are in interaction. Living systems can be represented as networks.[6,7]
Variations in hundreds of species evidenced laws of practical importance for agriculture and human health. Using log-log plotting we may evidence which of these laws are power laws.[7]

What operational definitions and general paradigms[8] can we use to build a useful model of living systems governance? What rules for their organisation[9] and survival can we evidence[10]?

2 Materials and methods

For every Living system, at every level of organization[10,11] (Figure 3), we can consider the paradigm of gauge invariance (Figure 2) as a system and study it in terms of interactions.[12]

![Figure 1. What a system is: the ecoexotope and endophysiotope model](image)

In Figure 1, a system is made of 3 kinds of entities: actors (color), their interactions (arrows) and their Whole (the system). Every system is a system-of-systems example of the cell[1] CC-License): the endophysiotope ENDO of a i level of organization (the hyaloplasm of the cell is the ecoexotope EXO of survival of previous i-n levels (the organelles: mitochondria, peroxisomes, plastids). Capabilities of previous levels are lost (actors half-autonomy), new ones are gained (recycling processes emergence). The new Whole is both less and more than of its parts.[1,4,13,14]

2.1 To quantify the qualitative interactions within the system

It is a usual approach in systems science, particularly in life sciences (Figure 4).[15] But it is not easy, because within every Living system there are 45 types of potent binary interactions, for at least 18 known levels of organizations (Figure 3), that means 810 bidirectional interactions. And these interactions need to be linked to each other at each stage of the organism development cycle (Figure 5) and at any level of organization.[13] This is not easy because, to contribute to the realization of a function, interactions can themselves be in interaction with other interactions (Figure 2). But it is possible to do so, even for organizations that are now missing.[13,16]

Is it possible to put the entire Universe in equations using the information available online? How to represent this network and model its evolution: by chance or by Laws?[17]

![Figure 2. The gauge invariance paradigm (Bricage’s diagram)[3,4]): every level of organization is defined by 7 functional capacities](image) In Figure 2, every capacity is both the result and at the origin of the other ones and itself. The capacities, mutually, modulate their expression. Arrows are indicating interactions: uni- and bi-directional (systemic constructal law (Figure 5) ones, that can act, separately or simultaneously, on capacities or interactions. The capacity of moving matter and energy flows is the first requirement, before the capacity of mass growth. The matter and energy flows and the growth are controlled through the capacity to respond to stimuli. All of that is allowed because the internal and external parts of the system exhibit a correlated structural and functional organization, into spaces,[11] through times[18] and interactions.[12,20] The external (EXO) and internal (ENDO) parts are not dissociable (Figure 1): integration. The capacities changes are continuously both the causes and the effects of the changes of each others. Continuously the capacity to be hosted HOSTED) of the ENDO is retroacting on the hosting capacity (HOSTING) of the EXO and reciprocally (systemic constructal law). Soon or late during its life cycle a living system -whatever its level of organization- expresses a capacity of movement. At least one time during its life cycle, into at least one
food chain. All capacities are mutually necessary and sufficient for the survival. The survival has only one goal reproduction of the life form: to have a offspring\cite{14} (CC-License).

2.2 What strategies with what graphic representation?

In humans physiology and psychology, measurements of reaction times (responses to stimulations) have evidenced functional laws designed by power functions\cite{12,21}. To adjust experimental data to a power law and statistically determine its exponent it is necessary to use a log-Log representation. This allows to have a line, which slope is the value of the exponent\cite{7,12,22}.

The use of a log-log scale helps to represent noncommensurable human-scale phenomena\cite{7} and functional structures, such as the structure of the stellar systems, e.g. the relationship between the mass of their star(s) and the distance of the planets to that star(s), in terms of zone of habitability (presence of liquid water). Not only the space scale (distance, surface and volume) but also the time scale must be represented in logarithmic coordinates (Figure 3). The difficulty is in the choice of appropriate units and experimental reference situations. For example, if $V_A$ is the volume of a living system, regardless of its level of organization, at the time it acquires its reproductive capacity, the adult state (i.e., the specialized stage of development for numerical growth), in cube meters, and if $t_g$ is the generation time, the time required to reach the threshold of the mass sufficient to acquire the reproductive capacity (i.e., the duration of the larval phase which is specialized for mass growth), in seconds, then, for the entire living systems of our Universe, all results experimentally obey a power law with a $3/2$ exponent\cite{7,12,16,20}, a line with a $3/2$ slope, with a probability of exceeding $90\%$, along $62 \times 62 \times 62$ dimensions of space and $62$ dimensions of time (Figure 3)\cite{7}.

Top in Figure 3: the periodic classification chart of living systems. Emerging from an ARMSADA each organization level is defined by its gauge invariance (Figure 2) Medium left: space-time-action relationship. The relationship $Y=f(X)$ between the adult volume $V_A$ of a system and its generation time $t_g$ is a fractal power law of slope $3/2$, with a scaling invariant growth curve, of logistic type (Medium right: growth) that can be linearized through log-log plotting. Down: a constant 2D interface flow. The $3/2$ coefficient is a hallmark of a Brownian mechanism of constant flowing at the surface, because $2/3 = 1/ (3/2)$ is the dimensional ratio of the surface (2D) to the volume (3D) of a sphere. (adapted from\cite{7,20,21} CC-License)

Figure 3. Living systems organization levels and growth limitation: juxtaposition and encasement is the process of systems emergence

2.3 A constraint: the reciprocity/reflexivity of interactions

We know other functional allometric relationships that are governed by power laws. For example, the relationship between the mass of a living system (the weight of its endophysiotope) and the extent of its hosting space (the surface or volume of its ecoexotope of survival), which is governing a quantitative relationship of interaction between the ecoexotope and the endophysiotope, the interaction between mass growth and number growth of the endophysiotope (Figure 6).

But there is no reason to tell that, a priori, it is the ecoexotope (by its limited capacity of hosting) that acts on the endophysiotope or that it is the endophysiotope (by its limited capacity to be hosted) that acts on the ecoexotope. At every time the endophysiotope and the
ecoexotope are in continuous multi-directional limited interactions, so we no longer know where the cause is or where the effect is, every actor is both cause and effect: systemic constructal law (Figure 5).

2.4 Biological meaning and fractional quantification

Let us look at the example of the Pieron’s law\(^{[12]}\) for the hearing, we can consider that everything happens as if the exponent of this power law is varying by jumps\(^{[12]}\). What are the possible exponents? What choice of discontinuity, what progression in continuous fraction, can we choose? Based on the results: 0, 1/2, 2/3, 3/4 (Kleiber’s Law\(^{[22]}\)), 1 (Weber’s Law), 4/3, 3/2 (Figure 7), 2, with the two series \(n/n+1\) and \(n+1/n\), both converging to 1, with an iterative emergent process (Figure 8) were chosen\(^{[5]}\) (Figure 9, Figure 10).

In Figure 4, rarely cause(s) and effect(s) relationships (Figure 5) are linear, but frequently laws are power laws that can be linearized with log-log plotting (-1-). Even we do not know which are causes or effects (systemic constructal law -2-) we can know what local actors are involved and what is depending/originating from ENDO or EXO. Often ENDO-EXO relationships obey a hyperbolic law, \(XY=K\), whatever causes or effects (-3-) ENDOxEXO=constant. YX=K laws in log-log plots give a line as power laws (-1-). Hyperbolic laws (-5-) like \(qQ=K\), where \(q\) is the ‘load’ of a local part and \(Q\) the global number of parts/actors (-4-), are hallmarks of functional limits and limitations; e.g. with \(q\) for quality, \(Q\) for quantity we get \((\text{quality})\times(\text{quantity})=\text{constant}\) a well known ecological and economical relationship. Power laws are hallmarks of fractal functional structures (Figure 6) \(^{[20]}\) CC-License.

In the relationship between \(VA\) and \(tg\) according to whether we consider mass growth as a cause or an effect we have a power law of exponent 2/3 or 3/2 (Figure 3), but growth is both a cause and an effect, and the two aspects are inseparable: the exponent, 3/2 or 2/3, corresponds to the same relationship of a functional, bi-directional, symmetrical interaction. As is, for example, the influence of growth in numbers (the density of a population of cells or organisms) on mass growth (or volume) and vice versa: \(3/2 = 1/(2/3)\) (Figure 6).

In Figure 5, every living system is a system-of-system, emerging by iteration of juxtaposition and embedment of previous ones Whatever the level, every endophysiotope \(i\) is both inhabiting an ecoexotope of survival and inhabited by other endophysiotopes \(i-j\) for which it is the ecoexotope of survival. Whatever the level defined by the gauge invariance characteristics (Figure 2), functions are interactions. So, 7 capacities means 45 different interactions. And with 18 levels of organization (Figure 3) that means 810 interactions! Rarely cause(s) and effect(s) relationships are linear, integration means sooner or later effects are causes and inversely, with simultaneous governmences: systemic constructal law (Figure 6)\(^{[5]}\) CC-License.)
2.5 Which results to be included in this meta-analysis?

How to validate the results to be included in the analysis? Most often it is mass, or mass growth, which is considered as the influencing factor (variable x). Let us take the example of a set of results relating to the allometric relationships of muscle energy.\[26,27\] What may the disparities between results to be due? Should we consider the most significant statistical relationship, for example an exponent \( \epsilon \) of -0.10 with a 95% statistical confidence, or the most significant physiological relationship, even if it is less statistically significant: \( \epsilon = 0.00 \) with 92% of statistical confidence? The biological significance must prevail over the statistical tool! One must always accept a slightly greater risk, a less statistical significant result, for a biologically very significant result. An exponent of 0.82 even at a 95% confidence with no biological meaning must be rejected, while the nearest exponent 0.75 (i.e., 3/4) at only a 93% confidence has a great biological significance.\[12\] So assessing the risk of error, a critical scientific mind must be maintained. Statistics must remain a tool, we must remain autonomous in understanding statistical results. Is it reasonable to mix phenomena at different levels of organization before taking into account interactions between levels of organization? It seems more reasonable to first evaluate interactions within a single level of organization\[22\] (Figure 6).

What unit to use to compare interacting phenomena at different dimensional scales\[21\]? What dimensional situation should be preferred? None! We must never give priority to our dimensional point of view, neither anthropomorphism, and, nor arbitrary scale...\[22-24\]

3 Results

In animals, many studies, at different levels of organization, have shown a link between the metabolic power and the mass of the producing system,\[16,20\] a power law of exponent 3/4 would report the same phenomenon, regardless of the level of organization. This law of Kleiber\[22\] would be valid on 27 dimensions, both for animals and plants.\[23\] Metabolism and growth would be dependent on the mass following a power law of exponent 3/4.

3.1 What is the link between energy production and mass?

For example, at the ecosystem level of a lake, the overall production of matter and energy by all autotrophic organisms (by photo-synthesis and chemo-synthesis) was measured. Then the consumption of the produced matter was measured by the respiratory activity of all heterotrophic organisms of the whole ecosystem food web.\[12\] The difference between production (power law of exponent 0.966 at 95%, with \( r=0.95 \) and p-value 0.00001) and consumption (power law of exponent 0.833 at 95%, with \( r=0.964 \) and p-value 0.0001) is compatible with the maintenance and growth of the whole ecosystem according to a power law of exponent 1/4 of the mass (1.00 - 0.25 = 0.75). The ecosystem is sustainable, able to survive and to save mass either in mass growth or number growth. The law of exponent 3/4 would be valid at this level of organization. Kleiber’s law would apply both to homeothermal organisms (mammals including humans, birds) and to exothermic organisms (or heterotherms) such as invertebrate animals. It was shown that solitary and colonial insects obey the same relationship between mass (of the individual or colony) and biomass production or consumption.\[12\] Mass production within the organism obeys a power law of exponent 0.74 (with \( r=0.99 \)) while consumption obeys a power law of exponent 0.81 or 0.82 (with \( r=0.91 \)), this along 7 dimensions of mass and 15 dimensions of metabolism. If this would be the true reality, no organism would be viable, because consumption would exceed production (0.74-0.81 = -0.07). Yet all exist! But, priority was given to statistics not to biological phenomena! And from one experience to another, the units were different..., so the comparison is impossible. Would this 3/4 power law, that has been taught for more than 50 years and validated by numerous scientific publications, be false? Where is the bias?

3.2 Would the observed reality adjust to a 2/3 power law?

From geometric and biological points of view, 2/3 is the hallmark of an optimized, spherical, interface for exchanging.\[27\] Indeed,\[20,21\] the relationship \( V / A = C . t g 3 / 2 \) (Figure 3, Figure 5, Figure 7) is the mark of a constant average flow of exchanges at the interface between ecoexotope and endophysiotope.\[1,5,6\] In fact, if we measure the basal metabolism by the metabolic consumption of oxygen (and no longer by the power produced, the heat released, or the thermodynamic entropy) we obtain power-laws with a 2/3 exponent.\[7,19\]

But it is true that it is difficult to separate 2/3 (or 0.66) and 3/4 (or 0.75) graphically. Statistical analysis does not necessarily help in interpreting the diversity of results (e.g. 0.70 + 0.10 or 0.70 + 0.05 or 0.65 + 0.03 ? with all at 95%\[8,12,13,16\]).

Before clustering them, organization levels should be analyzed separately.\[19\] The Whole is always different from the sum of its added parts. Before studying inter-
specific relationships, an ‘INTER’ variance, it is necessary to study intraspecific relationships, the ‘INTRA’ variance. Usually the INTRA variance is greater than the INTER variance. So, by individualizing organizational levels and relating them to the evolutionary rise in complexity of living systems,[7,11] let’s do a meta-analysis of results available online. Results are between 0 and +1 or between 0 and -1 (Figure 9). What does that mean from a functional point of view?

3.3 What about the regulation of matter and energy flows?

Whatever the organization level, the lowest variability is a hallmark of the inflows, INPUT, i.e. the system geometrically controls the inputs at its interface: incoming flows tend to be optimized at the interface of a spherical like volume (exponent 2/3). The outflows, OUTPUT, are either optimized (exponent 2/3) or controlled by feedback. A power law of exponent +1 is indicating a parallel evolution, both parameters are varying proportionately to each other, with inputs being proportional to outputs and vice versa, they are self-limiting (Figure 10).

What do mean power laws whose exponents are negative? A power law of exponent -1 is indicating aog-antagonism, with inverse changes of the parts, but a constance of the Whole: product XY=k (Figure 4, Figure 6). The internal circulating flows, THROUGHPUT, are of all kinds and show the greatest diversity (Figure 9). It is in accordance with the iterative processes of juxtapositions and embeddings (Figure 7) that give rise to new blue-prints, (Figure 3) by merging previous systems into a new ARMSADA.[3,5,10,11,21]

Every living system is a system-of-system, with half-autonomous compartmented parts.(Figure 1, Figure 7, Figure 8)

3.4 2/3 or 3/2, some aspects of a reflexive bidirectional arrow

The relationship between the growth in volume VA and the duration of the growth phase tg (i.e. the step before the acquisition of the adult reproductive state) is described by a power law of exponent 2/3, or 3/2, depending on which factor, VA or tg, we want to consider as influencing the other one.[7]

What does that mean?

One may consider either the temporal aspect, or the metabolic or energetic aspect, or the geometric or spatial (volume) aspect, they are exchangeable (Figure 7). A power law of exponent 2/3 is highlighting an optimized regulation, a minimal variation of fluctuations of a phenomenon, a least cost of functioning.[5]

Is there a physical process that can be at the root of that, spatial and temporal, fractal functional relationship?

3.5 Brownian motion is linking the parts and the Whole

If we look at a Paramecia, with an optical microscope, there is a permanent, chaotic agitation of microscopic grains in the water environment of its ecoexotope (Figure 1, Figure 5). Whether the organism is alive (with or without deadly coloring) or dead (the endophysiotope having been killed by a non-vital dye), this agitation does not affect the endophysiotope of the Paramecia. This random agitation process is the Brownian movement.[28] This process was elucidated by Francois Perrin, Albert Einstein and Robert Wiener.[29] In their model equation, time is affected by a 3/2 exponent, as it is in the relationship between VA (the adult mass volume at the end of the larval growth phase) and tg (the duration of this growth phase, i.e. the time for acquiring the capacity of reproduction, a critical step of the development cycle for a living system fate) (Figure 3). The relationship is true whatever the level of organization. Whatever the subpart of a system one takes, whatever is the part or the system, at every level of organization, even if we take the whole system-of-systems, and even the Universe as a Whole, this same invariant relationship governs the Whole, its parts, and the parts of its parts. It’s the fractal functional invariance of living systems.[3,11,21,30] The Universe itself appears as a sub-part, a sub-system, of a greater system-of-systems, a hyper-universe that hosts it, a Whole, that obeys the same relationship of fractal invariance (Figure 1, Figure 5).[7,16,19,27]

In Figure 6, mass growth M is a limiting factor of Number growth N. and reversely. Whatever X1=f(X2) or X2=g(X1) relationship(s), rarely cause(s) and effect(s) relationships are linear, but frequently they are power laws, laws that can be linearized with log-log plotting ([7,12] CC-License).

Brownian motion,[29] which is described by a 3/2 exponent power law of time, is itself an invariant fractal phenomenon.[7,20] The movements into the endophysiotope of an alive Paramecia, the local or global movements of cyclose, or all translation movements, are always oriented, well-ordered. The movements at the interface between the ecoexotope and endophysiotope are also, locally and globally, controlled, no chaos anywhere! And these inter-actions between movements and between space-time, and their co-interactions, are giving birth to rhythms, emergent properties are born, determinism is imposed on chaos.[5,19] So everything happens as if to the chaos of the ecoexotope of survival, to the
pre-existing indeterminism, the endophysiotope is imposing laws of order by increasing its spatial and temporal complexity.[15, 26, 31] It is the Taoist conception of the Universe, i.e. the existence of a system of laws, of interactions, regulates the harmonious functioning of the wholes, whatever the ecoexotope of survival, whatever the endophysiotope of growth and whatever the form of life we are looking at.[6, 19, 20, 32]

To define the complexity of a system, we must first, by deconstructing it, identify and characterize, qualitatively and quantitatively, all its parts. After reconstructing the whole from the parts, we talk of complexity if obviously we cannot explain new properties only by ancient previous ones: ‘a complex system is always more and less than the sum of its parts’. We can use 3 parameters to measure complexity: 1. action complexity, which is given by the numbers of each kind of actors, a colored point for everyone, all numbers will give the total number (we can define actors quantitatively by the surface of the points)-, or/and the numbers of each kind of interactions, a colored arrow for each kind, which all will give the total interactions number (we can also define interactions quantitatively by the thickness of the arrows). 2. time complexity, which is given by the duration of all the interactions that take place during a development phase (a time cycle of a step of survival). 3. space complexity, which is given by the absolute and relative spatiotemporal limits (interfaces) of embedded and juxtaposed food chains. To describe and explain why the set of actors, the set of interactions and the whole are making a stable system-of-systems, we can use either the actors or the interactions as ‘knots’ of differentiation, i.e. simultaneously as ‘cause and effect’ (systemic constructal law) in a markov web. More interactions taking place between actors involved in the same cycle, more complex is this cycle. More are pathways between actors involved in the same cycle, more are interactions and more complex is the whole. But evolution can increase or decrease complexity.[4, 8, 16] down: emergence is always a metamorphosis through a percolation process.

The emergence of a new system is a percolation process of cooperative interactions, it is a spatial-temporal and structural-functional meta-morphosis. The encapsulation and juxtaposition of the parts (Figure 1) obeys the same process, whatever the organization level. There are always 3 simultaneous processes in the metamorphosis: -lysis of ancient structures with the disappearance of previous actors during the interactive process of integration of at least 1 new actor, -creation of new functional structures, new actors that were not there before, are integrated into ‘the coming network’, -ancient actors are conserved but ‘transformed’ in their action, or in their place, or in their time of action (33) CC-License).

3.6 Man is not an exception

Man organism is not a physiological exception. The body wakefulness activity, the day, and the sleeping activity, at night, the night before and the night after each day, are linked together by a power law of exponent 3/2.[12] Man species is neither an ecological, nor an economic exception. Interactions between socio-economic indicators are often represented by a power law of exponent 3/2.[3, 16, 22] Man population is not a sociological exception too. The frequency of deaths is linked with the severity of attacks or technological risks[13] by a power law of exponent 3/2.[12]

4 Discussion and interpretation of results

With the molecular clock concept, it was assumed that the accumulation of mutations during evolution follows a linear timed process.[13] This is not the case: the accumulation of changes in amino acids in protein sequences,
follows a power law of exponent $2/3$.\textsuperscript{[1,5,7,12]}

### 4.1 Identifying limiting processes between factors in interactions

In a log-log plotting, a slope $\epsilon$ of +1 indicates that every parameter, $X_1$ or $X_2$, is proportional to the other one. This means that each of the observed factors, $x_1$ and $x_2$, obeying a power law of exponent +1, is a limiting factor of the other, in a bidirectional feedback process. If one increases, the other increases too. If one decreases, the other decreases too (Figure 9, Figure 10). Every one is both the regulating cause and the controlled effect of the other one (systemic constructal law (Figure 5, Figure 6)). Thus, both at the cellular and ecosystem levels, the metabolic rate is the limiting factor for mass growth, and vice versa, mass is the limiting factor for metabolic rate.\textsuperscript{[6,12,15,28]}

At the cellular and organelles levels, the spatial organization of the genome is depending on a process that limits the amount of information available (exponent +1).\textsuperscript{[12]} The processes of regulation of the access to the inherited information\textsuperscript{[32]} may change this relationship, making the space containing the genetic information more accessible (activation or de-repression: power law of exponent +5/4 (or 1+1/4)), or less accessible (repression: power law of exponent +3/4 (or 1-1/4)).

### 4.2 Highlighting invariant processes

A slope of 0 indicates an invariance, $x_1$ or $x_2$ are constant, regardless of $x_2$ or $x_1$ (Figure 10). For example, the number of lifelong heart beats in mammals is independent of the organism size, regardless of the lifespan. In vitro cultivation of cells frees them from the regulatory processes that take place within the organism of which they were parts. Their metabolism becomes invariant, independent of the cellular mass (slope 0), whereas it was negatively dependent on it in situ (slope -1/4).\textsuperscript{[12,22]} But the cells endophysiotope remains dependent on the ecoexotope conditions that are controlled by the experimenter.

### 4.3 Increase in space-time complexity is a rise in regulation

During the rise in organization emergence, the increase of the structural and functional, spatial and temporal complexity of living systems (Figure 8), from one level of organization to the next one (Figure 3), is the result of jumps.\textsuperscript{[1–3,12,19]}

From a situation of autonomy of the actors, exponent 0 (at the level of Monera, for example), to a situation of global no-freedom of the actors, exponent +1 (at the level of an ecosystem) or exponent -1 (Figure 9, Figure 10), jumps\textsuperscript{[3]} allow to increase the independence of the Whole. All actors of the endophysiotope are becoming increasingly dependent on each other at the same time as the endophysiotope becomes less and less dependent on the ecoexotope for its survival.

### 4.4 Regulation of matter and energy flows

When the ports of entry and exit, \textit{i.e.}, the ‘doors’ at the interface between the endophysiotope and ecoexotope, are different, \textit{reciprocal limitations} (exponent +1) can allow the compensation of inputs by outputs, and vice versa, which allows the Whole to maintain a constant volume, \textit{a steady state of maintenance}. What processes are represented by power laws with negative exponents (Figure 4, Figure 9, Figure 10)? $K= -1$ means that the 2 parameters vary inversely, proportionately to each other. The representative curve of their joint variation is \textit{an hyperbole}, their product $x_1.x_2$ is constant. The Whole is ‘the product’ of the parts.\textsuperscript{[33]} This indicates a reciprocal limitation of inflows and outflows, when inflows increase outflows decrease and vice versa. At different levels of organization, the fact that inputs exclude outputs and vice versa (exponent -1) is associated with the uniqueness of ports and interactions within the same port, and between ports at the endophysiotope-ecoexotope interface. This is the case for the relationship between mass growth and number growth (Figure 6), \textit{e.g.} for cells within an organism, or for organisms within an ecosystem, \textit{they exclude each other!} The existence of an intermediary situation, exponent +3/4 or +4/3 (Figure 9, Figure 10), may result from a situation of \textit{interaction between interactions}:\textsuperscript{[12]} $3/4 = 3/2 x 1/2$ or $4/3 = 2/3 x 2$, that is explaining why the previously observed Kleiber’s law could be a bias.
In Figure 7, cause(s) and effect(s) and their relationships are juxtaposed and embedded as spaces and times are (Figure 1, Figure 3, Figure 5). Even we do not know which are causes or effects (systemic constructal law) we can know what local actors are involved and what is depending/originating from either ENDO or EXO. Whatever causes or effects (Figure 4, Figure 5) in log-log plotting (Figure 3) we will get a line, which slope (Figure 3, Figure 4, Figure 6) is the exponent of reciprocal power laws. These power laws are the hallmarks of functional limits and limitation, either for time and reaction rates in metabolism or for space and the geometry of the half-autonomy and compartmentation (juxtaposition and embedment). Power laws are hallmarks of fractal functional structures \(^{[26]}\) CC-License).

4.5 Interactions between ecoexotope and endophysiotope

Variation of the interaction relationship, i.e. a change of the power law slope may indicate a pathological state or a de-regulation or dis-regulation state. In mammals, the frequency \((x1)\) and acceleration \((x2)\) of the heart rate are linked together by a relationship of the constant product type: \(x1 \times x2 = k\) (i.e. power law of exponent -1). After a heart transplant the relationship is changing (exponent is now -2) indicating another type of relationship, either a non-control or another control, between the hosted graft and the hosting organism.\(^{[7]}\) A physiological change or a change in the developmental stage, such as the transition from the larval stage to the adult one, with the acquisition of the reproductive capacity, may be associated with a slope break.\(^{[12]}\) Maintenance of the reproductive capacity has a metabolic cost, both structural and functional, which is paid by a flow of matter and energy that is no longer invested in growth. Before any statistical quantitative treatment, it is therefore important, to make first an eco-physiological\(^{[32]}\) qualitative analysis of the results. In industrial agriculture, an ecoexotope parameter, the temperature, has been extensively studied, in plants such as grasses (barley, wheat, corn\(^{[3]}\)). Temperature amounts or changes have different influences, depending on the physiological stage (the growth phase or the maintenance phase or the reproduction phase).

The exponent of the power law is indicating the effect of the temperature changes. So it is not enough to know the maps of the sum of temperature\(^{[5]}\) to predict the date of plant harvest. It is not the best statistical adjustment that should serve as a guide but the best physiological adjustment.\(^{[1, 2, 14, 16]}\)

4.6 From one step to another

Integration is depending both on age and stage of the actors, the interactions and the Whole (Figure 3). Connectedness in a network often shows a threshold behavior. When there are few connections, there are isolated islands of connections, and the largest connected group is a small fraction of total members in the network. However, at some point, the addition of a just a few more connections can cause a substantial fraction of the network to be connected.\(^{[7,12]}\)

Due to embedded levels of emergence,\(^{[12,19]}\) evolutive changes involve mosaic transformations, and ‘only instability, emergency, is an opportunity for emergence’. But, only a sufficient pre-requisite variety of species and levels of organization can generate, soon or late, more and more postpotential varieties of structures of the ARM-SADA type.\(^{[5]}\)

The (transient) stability is limited in time.\(^{[9,12]}\) It is born from the instability, more and more, and it generates, more and more, the instability. At all organization levels of the Universe, the growth (Figure 6) or the development of every living system are durable only if they are sustainable for their local partners and sustained by their local and global actions.

Figure 8. System complexity: the ARMSADA emergence percolation process

Top in Figure 8: definition and measurement of what
complexity is\(^{[3, 4, 18, 21, 24]}\) To define the complexity of a system, we must first, by deconstructing it, identify and characterize, qualitatively and quantitatively, all its parts. After reconstructing the Whole from the parts, we talk of complexity if obviously we cannot explain new properties only by ancient previous ones: ‘a complex system is always more and less than the sum of its parts.’ We can use 3 parameters to measure complexity: 1. **action complexity**, which is given by the numbers of each kind of actors, a colored point for everyone, all numbers will give the total number (we can define actors quantitatively by the surface of the points), or/and the numbers of each kind of interactions, a colored arrow for each kind, which all will give the total interactions number (we can also define interactions quantitatively by the thickness of the arrows); 2. **time complexity**, which is given by the duration of all the interactions that take place during a development phase (a time cycle of a step of survival); 3. **space complexity**, which is given by the absolute and relative spatiotemporal limits (interfaces) of embedded and juxtaposed food chains. To describe and explain why the set of actors, the set of interactions and the Whole are making a stable system-of-systems, we can use either the actors or the interactions as ‘knots’ of differentiation, i.e. simultaneously as ‘cause and effect’ (systemic contractual law) in a Markov web. More interactions taking place between actors involved in the same cycle, more complex is this cycle. More are pathways between actors involved in the same cycle, more are interactions and more complex is the Whole. But evolution can increase or decrease complexity.\(^{[4, 8, 16]}\)

Down in Figure 8: emergence is always a metamorphosis through a percolation process. The emergence of a new system is a percolation process of cooperative interactions, it is a spatial-temporal and structural-functional meta-morphosis. The encasement and juxtaposition of the parts (Figure 1) obeys the same process, whatever the organization level. There are always 3 simultaneous processes in the metamorphosis: 1. Lysis of ancient structures with the disappearance of previous actors during the interactive process of integration of at least 1 new actor; 2. Creation of new functional structures, new actors that were not there before, are integrated into ‘the coming network’; 3. Ancient actors are conserved but ‘transformed’ in their action, or in their place, or in their time of action \(^{[32]}\) CC-License.

5 Conclusion

Whatever the organization level, power laws are common in Living systems regulation, e.g. the mean and variance in cell number obey a power law with an exponent of 2, as does the Taylor’s law in ecological processes.\(^{[34]}\) Number or mass growth and ontogeny\(^{[35]}\) are associated with power laws obeying processes. Cities growth and innovation\(^{[36]}\) like cells growth and renewal\(^{[37]}\) obey power laws too.

Whatever the organization level, through the 18 organization levels of living systems-of-systems (Figure 3), 45 types of interactions for every endophysiotope (Figure 4), and at least 5 kinds of parameters (temperature, pH, radiation field, nutrients concentration, effectors concentration) which together define the hosting capacity of the ecoexotope, have to be measured.\(^{[12]}\) That is too much, thousands of interactions, to be able to fulfill, for all power laws, the table of the exponents that may allow to have a complete picture of the local and global functioning of all Living systems-of-systems. But it can be done for a limited number of interactions (Figure 9, Figure 10).\(^{[12]}\) This determinism in probability applies to populations, to large numbers, not to individual organisms and within biological and statistical limitations.\(^{[5, 8, 19]}\) But, using simulations, it is possible to describe a structural or/and functional state, and to predict an emerging state, a new state which structure is unknowable but which qualitative interactions between actors are knowable.

These interactions are quantitatively unknowable a priori, but will obey the same past and future laws of interactions. And this determinism is very prevailing since it allows living beings to shape their common ecoexotope of survival, within very strict limit “neither too much nor too little”,\(^{[1, 14, 15, 17, 37–40]}\) e.g. the shrimp larvae are so numerous and so active that they can have as much influence on the mixing of the ocean masses as the winds and swell.\(^{[2, 3, 7, 14]}\)

For every living system, at every level of organization,\(^{[11, 15, 17]}\) once the interactions are known and quantify, the ‘system of interactions’ (Figure 3) can be represented by a Markov network. The nodes of the graph are the functional capabilities and the oriented arcs are the probabilities of interactions (Figure 3, Figure 4). A Markov process is characterized by “a lack of memory”, the distribution of probability of next states depends only on the present state and not on past ones, it is “a contingency on the current context”. With a time interval equal to the generation time, i.e. t.g. over generations, sooner or later, a stationary state, a limit distribution, which does not depend on the initial situation,\(^{[40]}\) is achieved if the conditions of interaction, e.g. VA, between the ecoexotope and the endophysiotope remain unchanged.\(^{[20, 21]}\)

In Figure 9, power Law exponents \(\epsilon\) are: -1/2, -1/3, -1/4, -1/5, ..., -1/10 (the - 1/n series), 0, and +1/10, ..., +1/5, +1/4, +1/3, +1/2 (the symmetric + 1/n series), 1, and
Flows limits: determinism by laws and indeterminism by chance?

+2/3, +3/4, 4/5, +5/6, +6n, +7/8, +8/9, +9/10 (the n/n series), and 4/3, +3/2, +2 (the inverse n+ 1/n series), +5/2, 3 (according to the observed results). Exponents are relative only to the ‘body mass-metabolic rate’ relationship, but looking at metabolic compartmentation flows INPUT, THROUGPUT, or OUTPUT: local take-make-waste processes (Figure 2, Figure 7) without taking into account the organization levels. INPUT controls mainly obey power laws of exponents 2/3. OUTPUT obey 3 kinds of controls: power laws of exponents 2/3, 3/4, and 1. THROUGHPUT controls diversity is the hallmark of the variety of the organization levels we are looking at (from Monera 10 ecosystems (Figure 10), a data base with 432 papers).

The emergence of a new system (i.e. a new endophysiotope, integrated within a new ecoexotope or into a freely available ancient one) is unpredictable, both qualitatively, in its possible structure, and quantitatively, in terms of its latency time of emergence and its subsequent duration of survival. The interactions between endophysiotope and ecoexotope ‘sculpt’ the Nature fate and orient the evolution of the life forms that are sharing the same ecoexotope of survival. What are the factors of the ecoexotope that modulate the power laws that are representative of interactions within the endophysiotope? The increase in growth, either in volume, or in needed number of generations, is 2 to 8 fold faster in aquatic environments than in terrestrial ones. Growth reduction is 10 times faster in terrestrial environments. Gigantism increases faster in a marine environment. Dwarfism is typical of island spaces. This is consistent with the fact that the INTRA-level variance is always higher than the INTER-level variance. It is the rise in complexity of the web of interactions (Figure 8, Figure 9, Figure 10) that allows to cope with changes, both with a rise in complexity of organization levels and an increase in the resilience of living systems, which paradoxically become simultaneously more robust and more agile. They are more robust because they are increasing the capacity to be hosted (HOSTED) of their endophysiotope and more fragile when they over-increase the hosting capacity (HOSTING) of their ecoexotope of survival.

Power laws are hallmarks of the structures and rules (the grammar) of the living systems-of-systems common language and evolution: 1. invariant independent processes (power-laws with exponent \( e_0 = 0 \)); 2. regulation processes of simultaneous limiting interactions (exponent \( e_1 = +1 \)); 3. retro-action processes (\( e_2 = -1 \)); 4. competition between actors ago-antagonism (\( e_3 = 1/2 \)); and optimal exchanges flow (\( e_4 = 2/3 \)) (Figure 9, Figure 10). In the transition towards the emergence of an ARMSADA, that gives the new system simultaneously more autonomy for the Whole and more dependence between the actors, -that are jointly assembled into their Whole-, regulators are continuously adapting the structures through these power laws common rules. There are never advantages without disadvantages. Power laws relationships explain why ‘greater the advantages, greater the disadvantages’: more mass (M) for a stronger organism and a better individual survival (advantage) but less number of organisms (N) into the population and a weaker survival of the species (disadvantage), law \( MN = k \) (Figure 4, Figure 6). They explain why to survive that is to transform disadvantages into advantages and to avoid advantages turn to disadvantages, but within limits: “meden agan”. Man is not an exception.
In Figure 10, power Law exponents $\epsilon$ are the same as in Figure 9, the same results, but with no separation between INPUT, THROUGHPUT or OUTPUT, i.e. the global take-make-waste process of the Whole.\cite{12} The exponents are also related to the ‘body mass-metabolic rate’ relationship, but looking at the organization levels (Figure 3, Figure 5), only from Monera to ecosystems. There are never advantages without disadvantages. To survive and to itself survive its self, every living form must be integrated into an ARMSADA.\cite{1} An ARMSADA is the result of a synallagmatic deal between the systems of a system-of-systems. All partners are linked together for the best and for the worst.\cite{37} For a partner to survive, all others and the Whole must survive first.\cite{1} It is a network of networks with ago-antagonist feed-backs between actors. The hosting capacity of the ecxoecotope (HOSTING) and the capacity to be hosted of the endophysiotope (HOSTED) are linked together by a power law of exponent -1, $HOSTING\times HOSTED = k$, that allows the emergence, by percolation (Figure 8), and then the maintenance of the new system, regardless of the new integrated actors. By chance and laws Man is not an exception.\cite{2, 24}

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